THE POLYMER MODEL IN MRST

ABSTRACT. In this document, we describe briefly the polymer model which is implemented in MRST.

The `ad-core` module contains a solver for a black-oil system with polymer which includes shear-thinning effect. Here, we limit our presentation to the two-phase case and do not cover the shear-thinning effect.

Our starting point is the mass conservation equations for oil, water, and polymer

$$\frac{\partial}{\partial t}(\rho_\alpha \phi S_\alpha) + \nabla \cdot (\rho_\alpha \vec{v}_\alpha) = 0, \qquad \alpha \in \{w, o\}, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_w \phi S_w c) + \nabla \cdot (c \rho_w \vec{v}_{wp}) = 0. \tag{2}$$

Here, ρ_α, $\vec{v}_\alpha$, and S_α denote density, velocity, and saturation of the phase α. The porosity is denoted by ϕ and is assumed to be a function $\phi(p)$ of pressure only, c is the polymer concentration, and $\vec{v}_{wp}$ the velocity of water containing diluted polymer. Sources and sinks may be included in a manner equivalent to boundary conditions, and are left out of the above equations.

To model the viscosity change of the mixture, we use the Todd–Longstaff model [1]. This model introduces a mixing parameter $\omega \in [0, 1]$ that takes into account the degree of mixing of polymer into water. Assuming that the viscosity μ_m of a fully mixed polymer solution is a function of the concentration, the effective polymer viscosity is defined as

$$\mu_{p,\text{eff}} = \mu_m(c)^\omega \mu_p^{1-\omega} \quad \text{with} \quad \mu_p = \mu_m(c_{\max}). \tag{3}$$

The viscosity of the partially mixed water is given in a similar way by

$$\mu_{w,e} = \mu_m(c)^\omega \mu_w^{1-\omega}. \tag{4}$$

The effective water viscosity $\mu_{w,\text{eff}}$ is defined by interpolating linearly between the inverse of the effective polymer viscosity and the partially mixed water viscosity

$$\frac{1}{\mu_{w,\text{eff}}} = \frac{1 - c/c_{\max}}{\mu_{w,e}} + \frac{c/c_{\max}}{\mu_{p,\text{eff}}}. \tag{5}$$

To take the incomplete mixing into account, we introduce the velocity of water that contains polymer, which we denote $\vec{v}_{wp}$. For this part of the

water phase, the relative permeability is assumed to be equal to k_{rw} and the viscosity is equal to $\mu_{p,\text{eff}}$. We also consider the total water velocity, which we still denote $\vec{v}_w$ and for which the viscosity is given by $\mu_{w,\text{eff}}$. Darcy's law then gives us

$$(6) \qquad \vec{v}_w = -\frac{k_{rw}}{\mu_{w,\text{eff}}R_k(c^a)}\boldsymbol{K}(\nabla p - \rho_w g \nabla z),$$

$$(7) \qquad \vec{v}_{wp} = -\frac{k_{rw}}{\mu_{p,\text{eff}}R_k(c^a)}\boldsymbol{K}(\nabla p - \rho_w g \nabla z) = m(c)\vec{v}_w,$$

as we assume that the presence of polymer does not affect the pressure and the density. The polymer mobility factor $m(c)$ is defined as

$$m(c) = \frac{\mu_{w,\text{eff}}}{\mu_{p,\text{eff}}}$$

and, after some simplifications, we get

$$(8) \qquad m(c) = \left[\left(1 - \frac{c}{c_{\max}}\right)\left(\frac{\mu_p}{\mu_w}\right)^{1-\omega} + \frac{c}{c_{\max}}\right]^{-1}.$$

The function $R_k(c^a)$ denotes the actual resistance factor and is a non-decreasing function which models the reduction of the permeability of the rock to the water phase due to the presence of absorbed polymer. The concentration of absorbed polymer is denoted by c^a. We introduce the total flux as $\vec{v} = \vec{v}_w + \vec{v}_o$. We have

$$\vec{v} = -(\lambda_w + \lambda_o)\boldsymbol{K}\nabla p + g(\lambda_w \rho_w + \lambda_o \rho_o)\boldsymbol{K}\nabla z$$

and, after some computation, we obtain the following expression of $\vec{v}_\alpha$ as a function of $\vec{v}$

$$(9) \qquad \vec{v}_w = f_w \vec{v} + \vec{v_g} \quad \text{and} \quad \vec{v}_o = f_o \vec{v} - \vec{v_g}$$

with

$$(10) \qquad \vec{v_g} = \frac{\lambda_w \lambda_o}{\lambda_w + \lambda_o}(\rho_w - \rho_o)g\boldsymbol{K}\nabla z.$$

Here, λ_α denotes the mobility of phase α, i.e.,

$$\lambda_w = \frac{k_{rw}}{\mu_{w,\text{eff}}R_k(c^a)} \quad \text{and} \quad \lambda_w = \frac{k_{ro}}{\mu_o},$$

and f_α corresponds to the fractional flow, $f_\alpha = \lambda_\alpha/(\lambda_w + \lambda_o)$. The time scale of adsorption is much larger than that of mass transport and we will assume that adsorption takes place instantaneously so that c^a is a function of c only. The reference rock density is $\rho_{r,\text{ref}}$ and the reference porosity ϕ_{ref}. The adsorption of polymer is then taken into account by replacing (2) by

$$(11) \qquad \frac{\partial}{\partial t}(\rho_w \phi S_w c) + \frac{\partial}{\partial t}\left(\rho_{r,\text{ref}}(1 - \phi_{\text{ref}})c^a\right) + \nabla \cdot (c\rho_w \vec{v}_{wp}) = 0.$$

It is natural to assume that c^a is an increasing function of c. Polymer cannot reach the smallest pores and, as a result, the effective pore volume for the

polymer solution is smaller than the pore volume of the rock. This effect can be modeled by replacing (11) with

$$(12) \quad \frac{\partial}{\partial t}(\rho_w \phi(1 - S_{\mathrm{dpv}})S_w c) + \frac{\partial}{\partial t}(\rho_{r,\mathrm{ref}}(1 - \phi_{\mathrm{ref}})c^a) + \nabla \cdot (c\rho_w \vec{v}_{wp}) = 0.$$

where S_{dpv} denotes the fraction of the pore volume which is not accessible to polymer. The introduction of dead pore volume has the effect to increase the mobility of the polymer solution. However, the model equation (12) yields to instabilities because it allows polymer to go faster than its solvent (water). Actually a consistent way to introduce dead pore volume may be given by replacing the definition of the effective water viscosity given by (5) by

$$(13) \qquad \frac{1}{\mu_{w,\mathrm{eff}}} = \frac{1 - c/c_{\max}}{\mu_{w,e}} + \frac{c/c_{\max}}{(1 - S_{\mathrm{dpv}})\mu_{p,\mathrm{eff}}}.$$

In this case, the polymer mobility factor becomes

$$(14) \qquad m(c) = \frac{\mu_{w,\mathrm{eff}}}{\mu_{p,\mathrm{eff}}} = \left[\left(1 - \frac{c}{c_{\max}}\right) \left(\frac{\mu_p}{\mu_w}\right)^{1-\omega}(1 - S_{\mathrm{dpv}}) + \frac{c}{c_{\max}} \right]^{-1}.$$

Finally the modeling equations are

$$(15\mathrm{a}) \qquad \frac{\partial}{\partial t}(\rho_\alpha \phi S_\alpha) + \nabla \cdot (\rho_\alpha \vec{v}_\alpha) = 0,$$

for $\alpha \in \{w, o\}$,

$$(15\mathrm{b}) \qquad \frac{\partial}{\partial t}(\rho_w \phi S_w c) + \frac{\partial}{\partial t}(\rho_{r,\mathrm{ref}}(1 - \phi_{\mathrm{ref}})c^a) + \nabla \cdot (c\rho_w \vec{v}_{wp}) = 0.$$

where $\vec{v}_\alpha$ and $\vec{v}_{wp}$ are defined in (6) and (7) using (3), (4) and (13).

References

[1] M. R. Todd and W. J Longstaff. "The Development, Testing, and Application Of a Numerical Simulator for Predicting Miscible Flood Performance". In: *J. Petrol. Tech.* 24.7 (1972), pages 874–882.