Turbulent and financial time series analysis

Amjed Mohammed (amjed.a.mohammed@uni-oldenburg.de)

Abstract: Some of the characteristics of turbulence are its randomness, nonlinearity, diffusivity, and dissipation, just to name few. But couldn't we characterize financial data in the same way? The answer is no, not exactly. Some of the extra descriptions for financial data, which makes it different than the steady experimental turbulence, are its Markovity and non-stationarity.

Turbulent signals:

Fig.(1) shows a non-trended noncompressible stationary turbulent velocity signal, measured in an airtank experiment in Oldenburg university. The spectrum of turbulent signals was shown by Kolmogorov to be equal to :

$$E(k) = C\epsilon^{2/3}k^{-5/3}$$

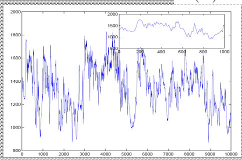

Fig. 1

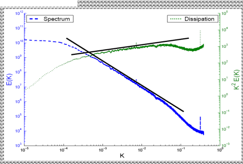

Fig. 2

where E(k) is the spectrum as a fuction of the wavenumber, C is Kolmogorov's constant, ε is the dissipation rate, and K is the wavenumber. We see clearly that the slope of the spectrum represented by the blue color in fig. (2) is equal to -5/3, while the green color represents the dissipation spectrum and its slope equals 1/3, which confirms the theory. In addition to that, the energy spectrum shows three distinct regions, namely, the large scales, the inertial range, the dissipation range and the random region. Taking a look at figs (3) and (4) we see that the same same three regions are more or less represented. First comes the Taylor microscale (λ) which is the curvature of the autocorrelation which is approximately equal for both data sets. Fig(3) data set is the same as in fig.(1) above and in fig.(4) we show another data set with a higher Reynolds number (Re=UL/ν) . We notice that in both cases the autocorrelation is finite and the zero crossing is the beginning of the random region. Fig.(3) shows the phase diagram or the bivariate probability density function of the high Reynolds number data and we notice the Gaussian mexican hat upon reaching the zero crossing point.

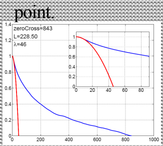

Fig.3

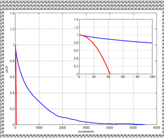

Fig.4

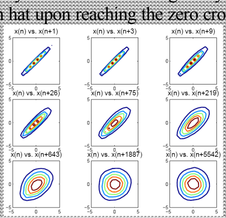

Fig.5

A useful tool in studying turbulence is the structure function or the increment and is equal to δu=|u(x+r)- u(x)|, where u is the velocity at position x, and r is the lag. The higher order structure functions which equals:

$$\langle|u(x+r)-u(x)|^p\rangle = C_p(\langle\epsilon\rangle\ r)^{p/3}$$

describes the cascading of the energy from large scales to small scales. This cascading follows a power law as is shown in the following figures.

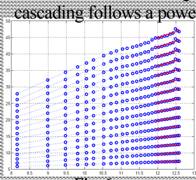

Fig.6

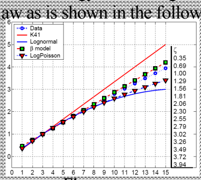

Fig.7

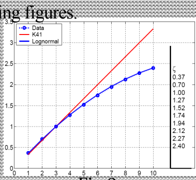

Fig.8

In fig.(6) the structure functions till the exponent p=15 were calculated and in fig.(7) we see the scaling of these structure functions according to

$$S_p(r) \sim r^{\zeta_p}$$

In fig.(7) we have used the extended self-similarity to show the power law scaling of the structure functions. We see also that the best fit is Kolmogorov's lognormal scaling model which is

$$\zeta_p = \frac{1}{3}p - \frac{1}{18}\mu\ p(p-3)$$

In fig.(8) the dissipation was fitted with the lognormal and by tunning the parameter μ one could find the best fit which is 0.24.

The question now i whether the above tools are suitable for non-stationary time series like the global warming temperatures. In figs. (9), (10), and (11) we show the temperature time seriese, the detrended temperature and the incremented temperature.

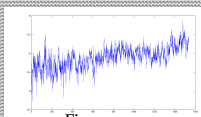

Fig.9

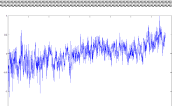

Fig.10

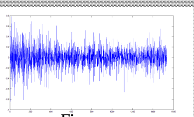

Fig.11

Financial signals:

Financial time series are non-stationary, i.e. the moments are a function of time. This is evident from fig.(12a), which is for the DAX index for the period from 16.2 till 31.12.2001. In (b) the autocorrelation function which shows a long memory. In (c) we see the spectrum which scales as -2 and not -5/3 as in the turbulent, while the probability density functions (PDFs) show anomalous scaling first and by increasing the lag they reach probably, one could say, a uniform distribution form upon reaching the zero crossing point.

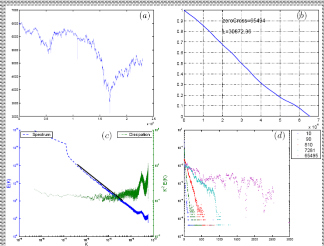

Fig.12

Non-stationary time series are modelled by a Wiener process:

$$x_t = x_{t-1}\ +\ \mu\ t\ +\ \sigma\ \eta\ \sqrt{t}$$

where x is a stochastic variable, μ is the mean (trend) of the process,σ is the variance (volatility), η is random noise, and t is the time. In fig.(13a) we show such a time series, in (b) its autocorrelation, in (c) the spectrum with two slopes -2 and -5/3 and in (d) the PDFs.

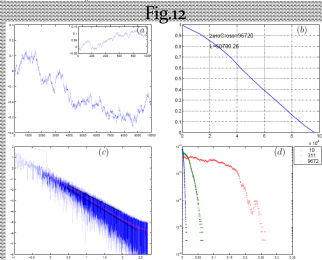

Fig.13

An important result from the above is that the tools that were used to analyze stationary turbulence are not helpful in analyzing non-stationary data, were this is evident from the plunge of the DAX data on 11.9.2001 and the spectra of both processes still show a (-2) slope.

An important tool to see the content of the spectrum of a signal in time and Fourier space is the spectrogram. In fig. (14) we see the spectrum of a sinusoid signal with two frequencies entrupted by a random band in two places, but the spectrum shows only the two frequencies. In fig. (15) we used a spectrogram to show the frequencies on the y-axis and the x-axis shows the interuption bands. Another tool is the Wigner–Ville spectrum which again shows both domains the frequency on the y-axis and time on the x-axis. Here we have used a noisy sinusoid interupted by two bands of noise. The need for other tools arises because the structure functions (the return) gives simply a random process.

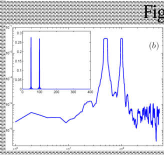

Fig.14

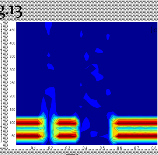

Fig.15

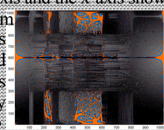

Fig.16

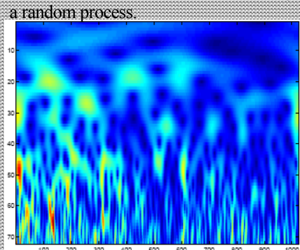

Fig.17

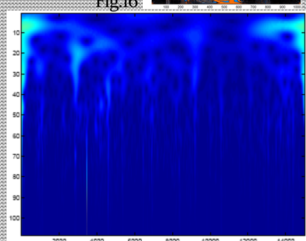

Fig.18

At last we show in figs.(17) and (18) a wavelet analysis for the signals that appeared in figs. (9) and (12a) respectively.

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