Feature based visualization
Topology in Visualization – An Introduction

January 2016

Geilo winter School – Scientific Visualization

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LINKÖPINGS UNIVERSITET

I Topological methods for visualization - Introduction

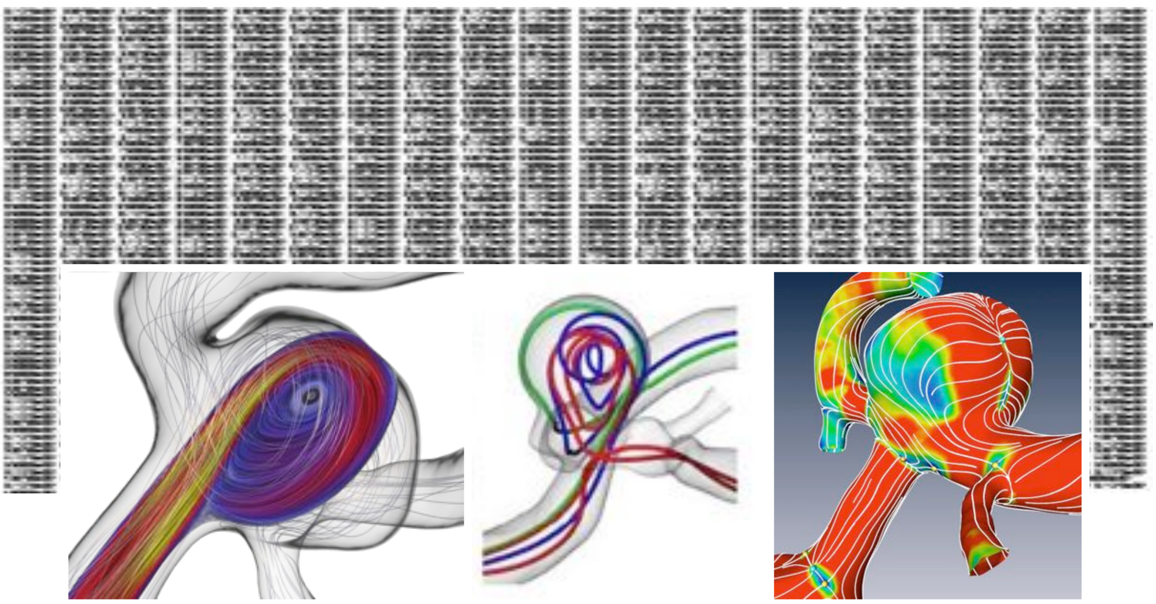
Visualization Center Norrköping

Overview

I. Introduction
   1 Feature based visualization
   2 Topology – a mathematical discipline
   3 Topology – a concept for data analysis
II. Scalar field topology
III. Vector field topology
IV. (Tensor field topology)

1 Feature based visualization

One (a few) image(s) say(s) more than a thousand words (numbers)



Images: Petz, Zuse Institute Berlin (ZIB) , Amira

1 Feature based visualization

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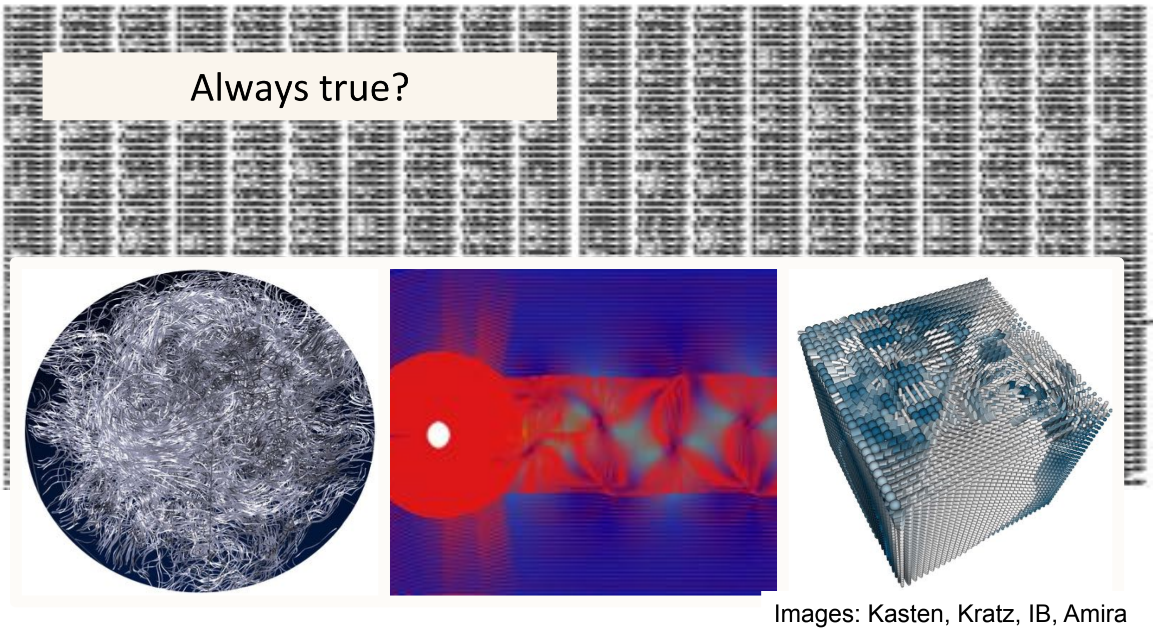


Images: Kasten, Kratz, IB, Amira

1 Feature based visualization

Challenge

- Too much and/or too complex data to be shown all at once
- Find the "important"
- Need for **explicit structures** that can be used for further analysis

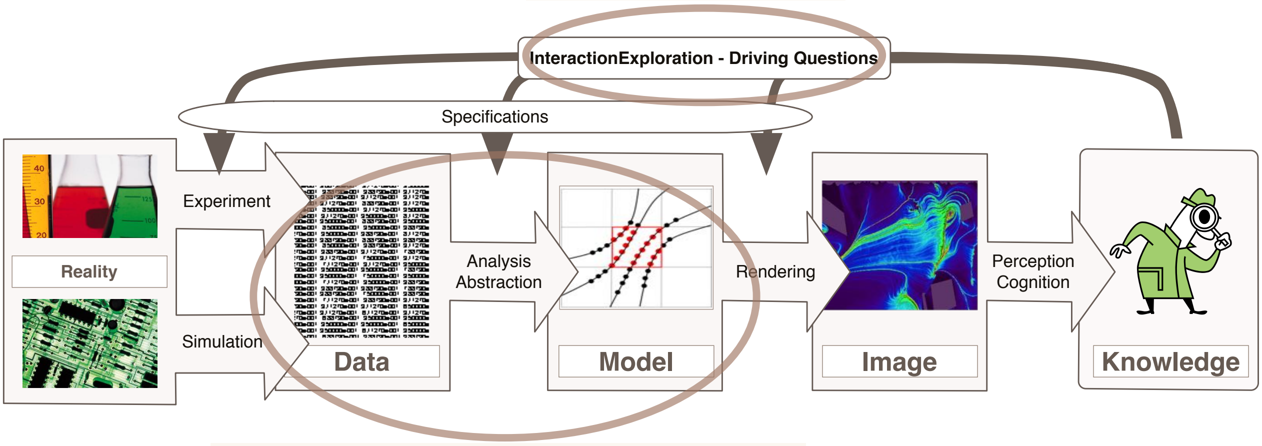
Feature based visualization

- Data reduction tailored to specific needs, **questions or interests**
- Extraction of **structure, whatever this means**

There are many steps involved from feature definition to extraction

1 Feature based visualization

Specific to questions



Complexity reduction
Concepts – Techniques – Algorithms

1 Feature based visualization

Example – flow visualization

Task: Impart knowledge

- Telling stories
- Presentation

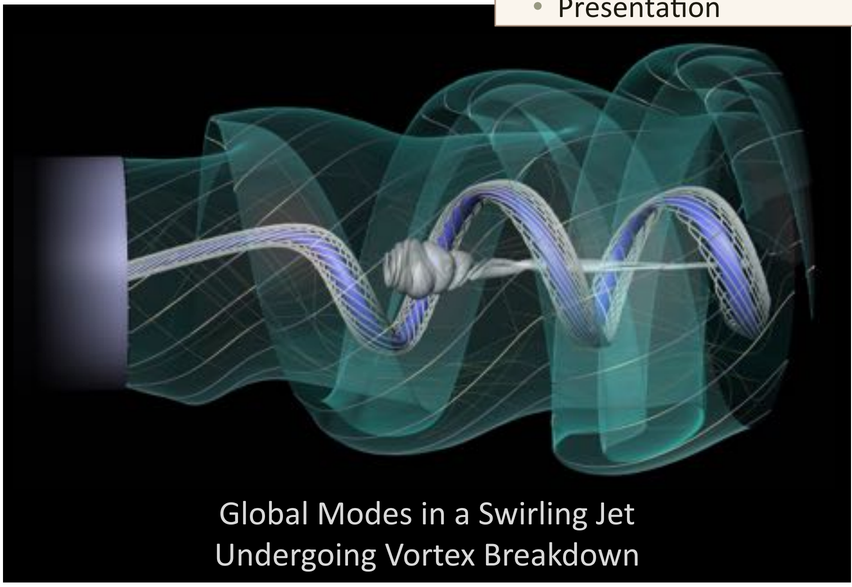


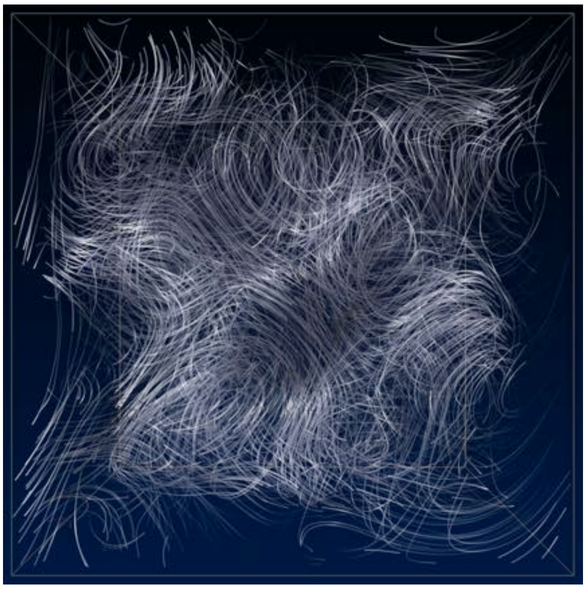
Image: Petz, ZIB, Amira

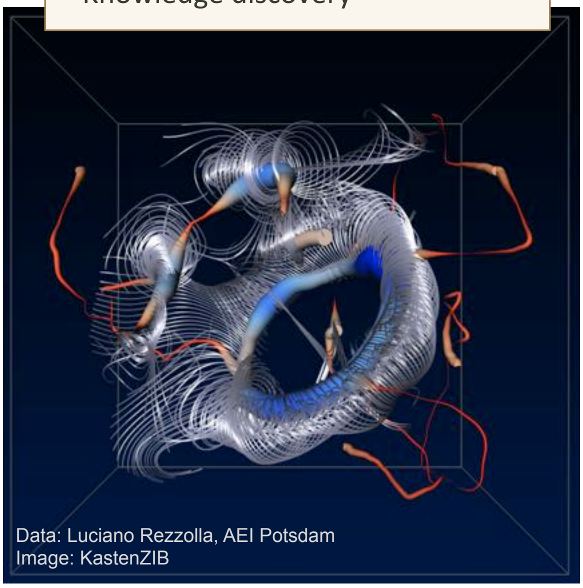
1 Feature based visualization

Example – relativistic hydrodynamics

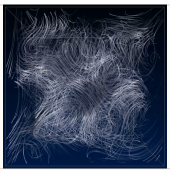
Task: Understand and analyze

Knowledge discovery





1 From **Feature** Definition to Extraction

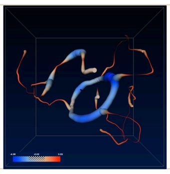


1. Feature Identification
   – Intuitive notion
   E.g. vortex structures

→ Domain Specific Guidelines
E.g. Some invariance

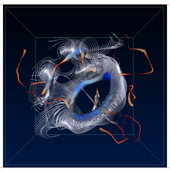
2. Mathematical Description
   – Precise definition
   E.g. maxima in vorticity field

→ Mathematical Framework
E.g. Scalar field topology



3. Extraction from the Data
   – Algorithmic realization

→ Algorithmic Guidelines
E.g. Robustness against noise



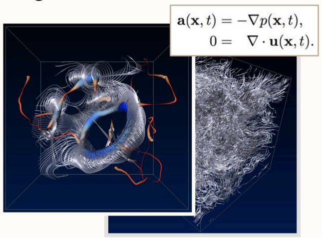
4. Visual Representation and Verification

→ Visualization Guidelines

1 Feature based visualization – possible views

Closed Form Feature Definition
target known

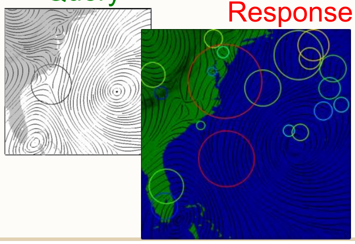
E.g. vortex extraction



Feature as Pattern
Visual Selection

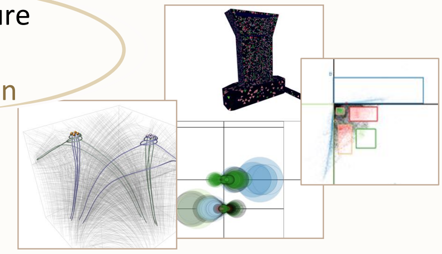
Query

Response



Explorative Feature Definition
target not known

E.g. tensor field analysis



1 Motivation

Challenge

- Too much and/or too complex data to be shown all at once
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- Need for **explicit structures** that can be used for further analysis

Feature extraction

- Data reduction tailored to specific needs, **questions or interests**
- Automatic extraction of **structure**

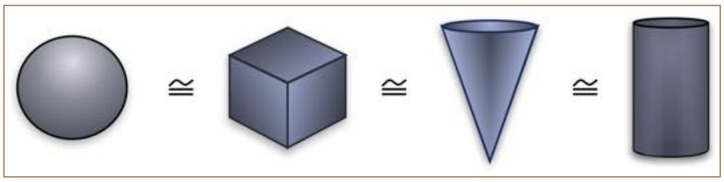
Topology provides one way to approach this challenge

2 Topology – a mathematical discipline

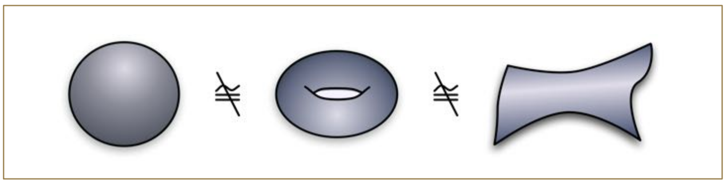
2 Topology – a mathematical discipline

What is topology? – Intuition

- Greek: topos - place, logos – study
- "Rubber sheet geometry"
- "Qualitative Geometry"
- Study of shape properties **invariant under continuous deformation**



- No cutting
- No merging
- No poking
- No sealing holes



2 Topology – a mathematical discipline

Topological spaces – Definition

Definition

A ***topological space*** is

- **a set *X*** together
- collection of **subsets** of *X* (**open sets)**

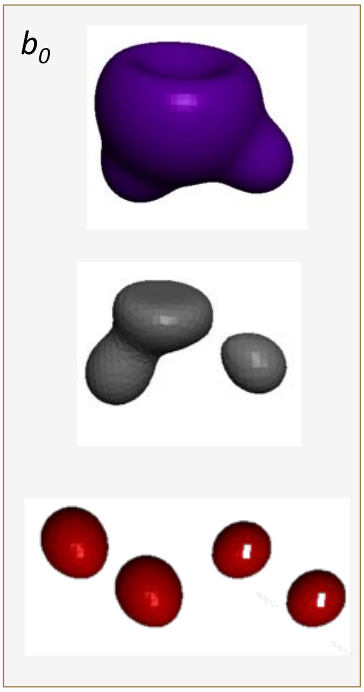
Satisfying some **axioms**

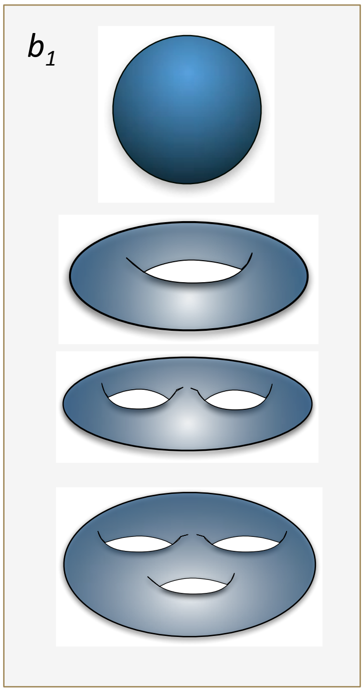
- Refers to concepts of **neighborhood**, continuity, connectedness, convergence
- Defines which points **are near each other without specifying the distance between them**.

2 Topology – a mathematical discipline

Algebraic Topology – **Homology**

Analyzes manifolds according to **topological invariants**





Betti Numbers

b_0: Number of components

b_1: Number of tunnels

b_2: Number of voids

….

2 Topology – a mathematical discipline

Selected spotlights in context with data analysis

- **Morse theory** establishes a link between differentiable function on manifolds and the manifold's topology
- **Topological dynamics** studies qualitative, asymptotic properties of dynamical systems from the viewpoint of general topology
- **Computational topology** deals with practical solutions for solving topological problems developing efficient algorithms

2 Topology – a mathematical discipline

Literature – introduction to algebraic, computational topology

- Allen Hatcher, *Algebraic Topology*, Camebridge, 2002
- Herbert Edelsbrunner: *Geometry and topology for mesh generation*
- H. Edelsbrunner, J. Hare: *Computational Topology An Introduction*, American Mathematical Society, 2010
- J. J. Sánchez-Gabites, *Dynamical systems and shapes*, RACSAM: Geometry and Topology, 2008
- R Forman, *Morse Theory for Cell Complexes*, Advances in Mathematics, 1998
- R Forman, *A user's guide to discrete Morse theory,* Applied Mathematics, 2001

3 Topology – a concept for data analysis

1 Motivation

Challenge

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- Need for **explicit structures** that can be used for further analysis

Feature extraction

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Topology provides one way to approach this challenge

3 Topology – a concept for data analysis

- Brings structure in the data
- Provides a summary of shape and field properties
- Many applications in Visualization:
  - Iso-suraface characterisation
  - Extremal structure extraction
  - Feature preserving smoothing - simplification
  - Segmentation – scalar, vector, and tensor fields
  - Skeleton computation
  - Mesh generation from point samples
  - …….

3 Topology – a concept for data analysis

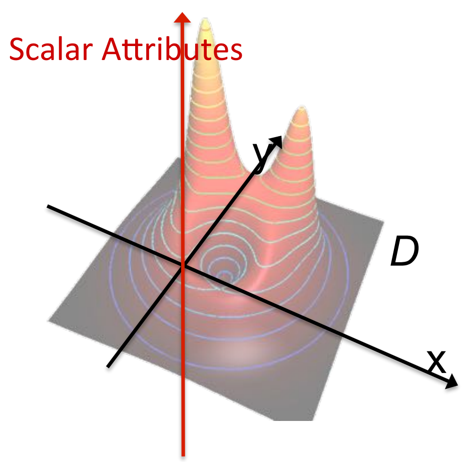
Example

Scalar field visualization

Recall - Contours (isosurfaces)

Set of points to given scalar value $w \in R$

$$\{(x,y) \in D \mid s(x,y) = w\} = s^{-1}(w)$$



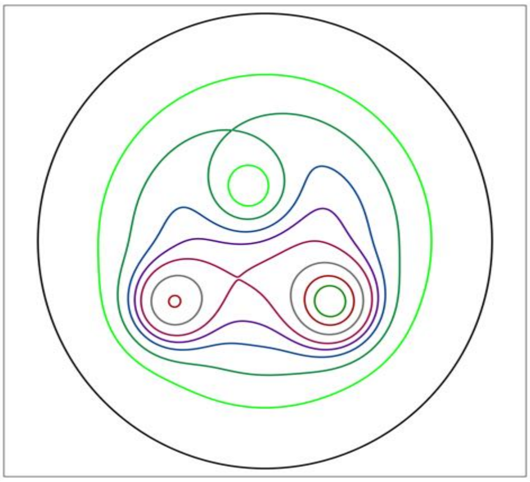
Height field + contours

3 Topology – a concept for data analysis

Example

Scalar field visualization

Contour Tree
- **Equivalence classes** for contours



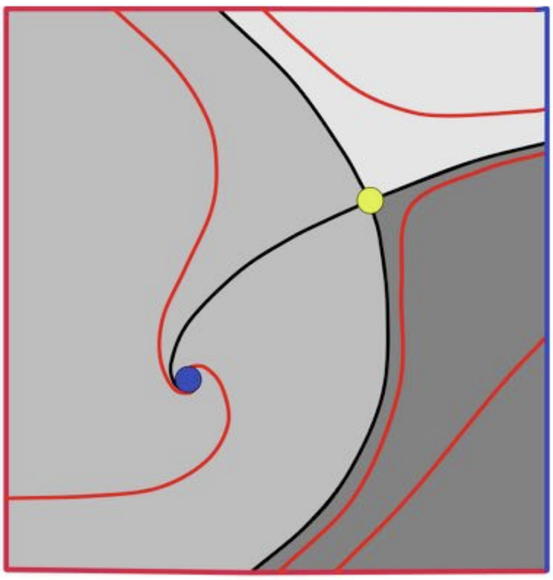
I Topological methods for visualization - Introduction

3 Topology – a concept for data analysis

Example

Vector field visualization

Topological graph
- **Equivalence classes** for streamlines

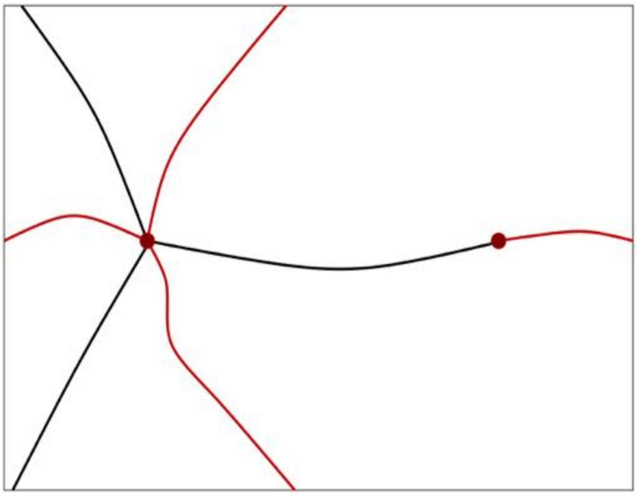


3 Topology – a concept for data analysis

Example

Tensor field visualization

Topological graph
- **Equivalence classes** for tensorlines



3 Topology – a concept for data analysis

Subfields

- Scalar field topology → computational discrete Morse theory
- Vector field topology → dynamical systems
- Tensor field topology

These are related topics, however have different focus in research and applications

Overview

I. Introduction

II. Scalar field topology

1. **Contour tree**
2. **Critical points**
3. **Morse Smale complex**
4. **Extremal structures**
5. **Simplification**
6. **From analytical concepts to discrete realizations**
7. **Examples from flow visualization**

III. Vector field topology

IV. Tensor field topology

2 Contour tree

2 Contour tree

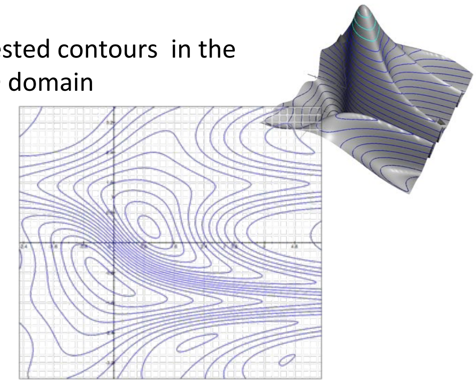
Contours or isosurfaces
(also **level-sets**, or **implicit surface**)

Set of points to given scalar value *w* ∈ *R*

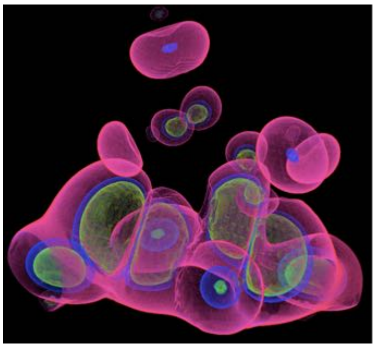
$$\{(x,y,z) \in D \mid s(x,y,z) = w\} = s^{-1}(w)$$
$$D \subset \mathbb{R}^3$$

One of the major challenges:
- Is there a set of iso-values that provides a **'complete' picture** of the data?
- If yes how can we find it?

Nested contours in the 2D domain



Nested isosurfaces in a 3D volume



2 Contour tree

Contours or isosurfaces
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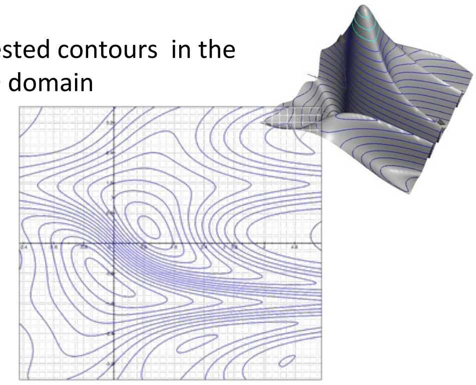
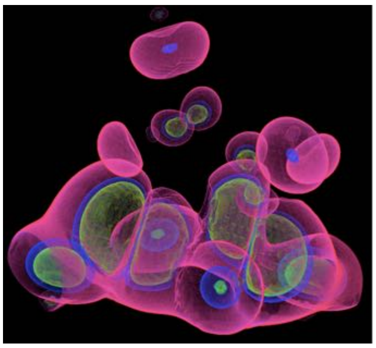
$$D \subset \mathbb{R}^3$$

Note: In the following we use the terminology independent from the dimension:
- **Level set** for $s^{-1}(w)$
- **Contour** for one connected component of $s^{-1}(w)$

Nested contours in the 2D domain



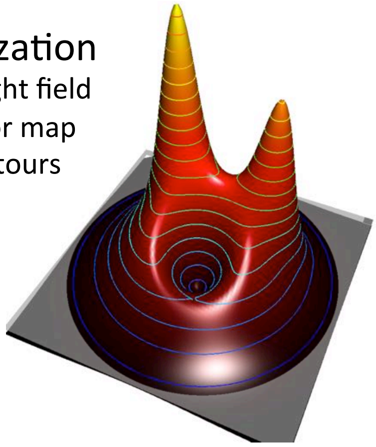
Nested isosurfaces in a 3D volume

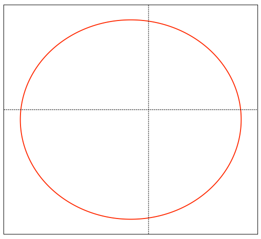


2 Contour Tree - Illustrative Example

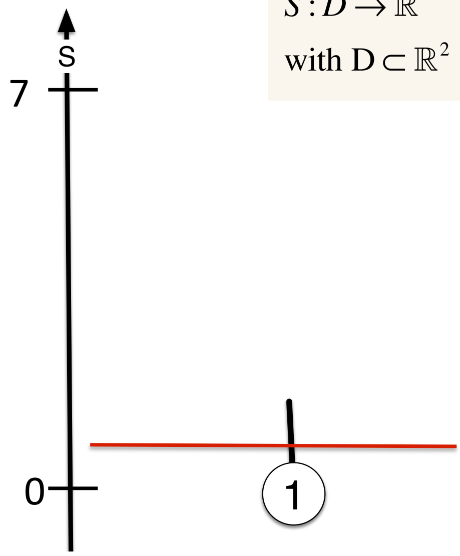
Example:
2D Scalar function.
$S : D \rightarrow \mathbb{R}$
with $D \subset \mathbb{R}^2$

Visualization
- Height field
- Color map
- Contours

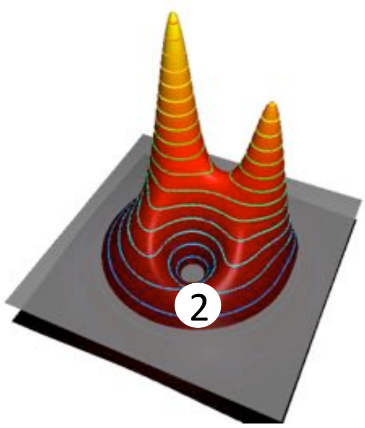


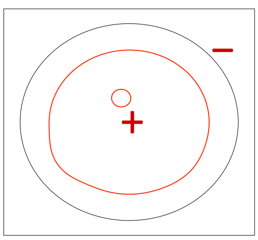


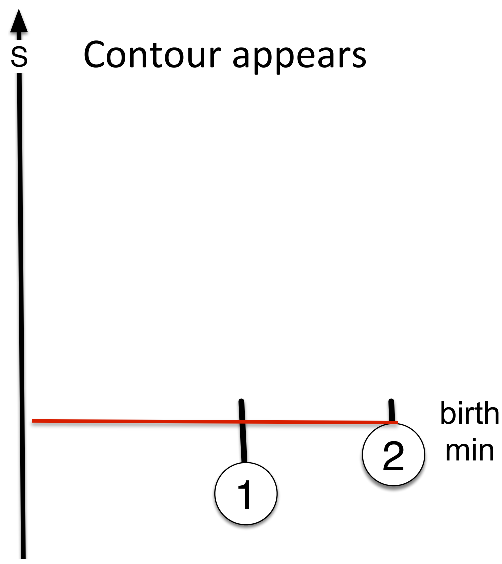
Scalar Attributes



2 Contour Tree - Illustrative Example







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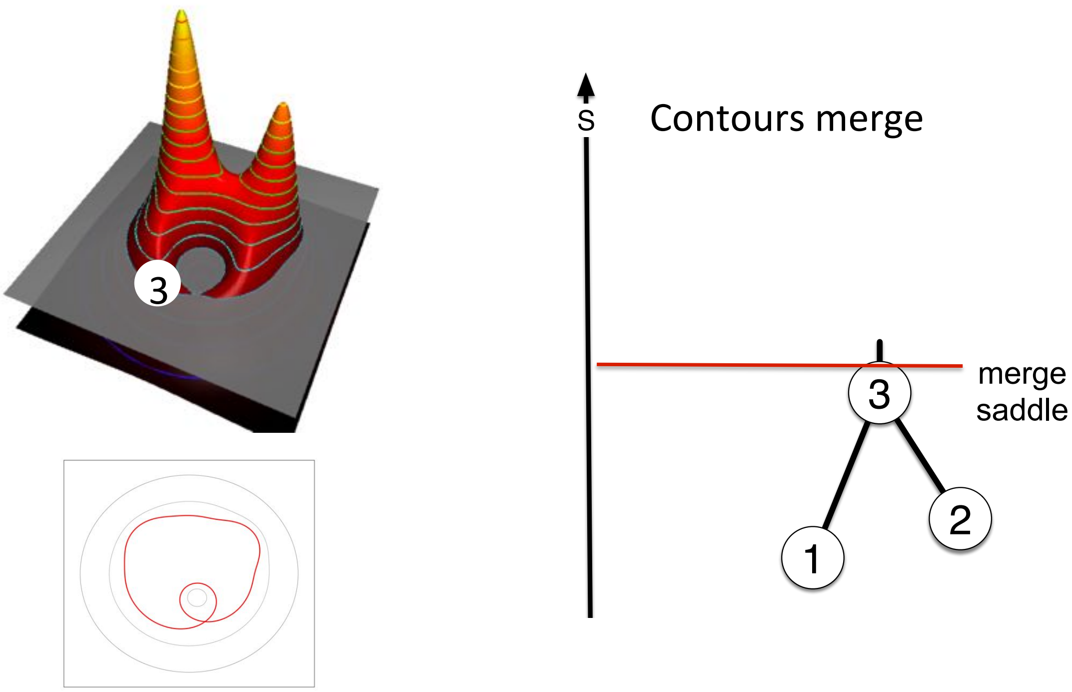

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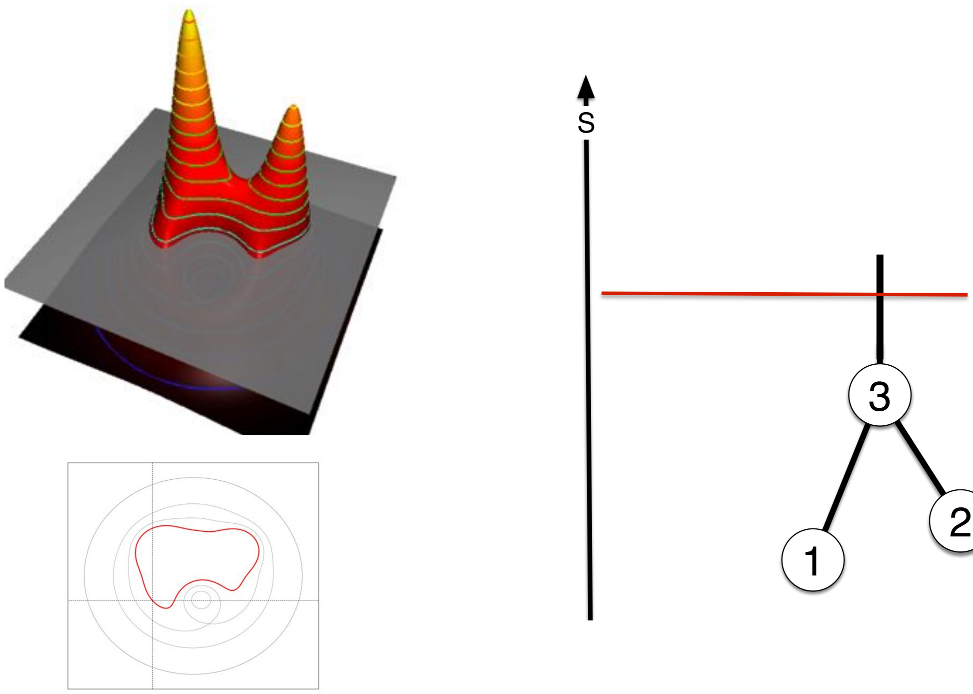

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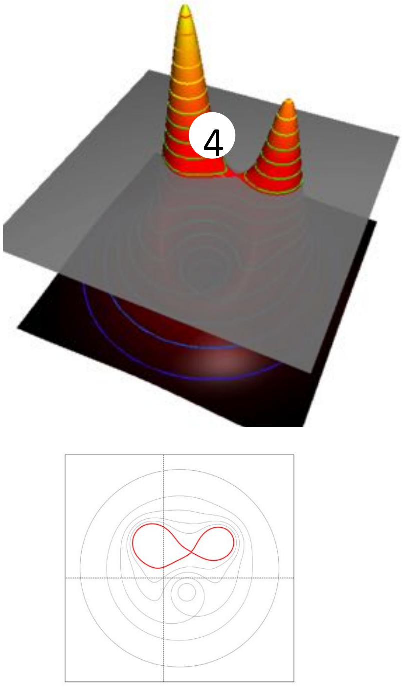
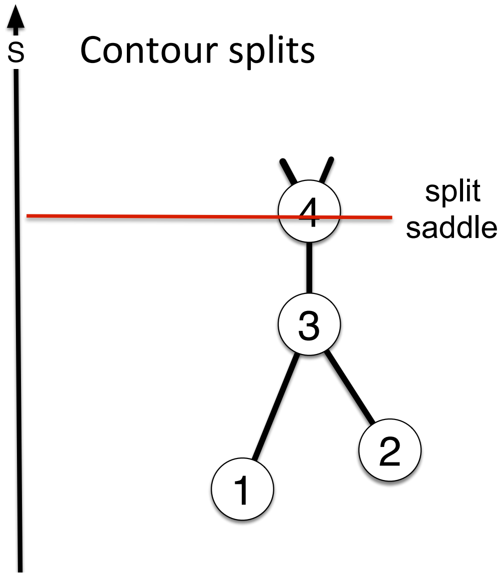
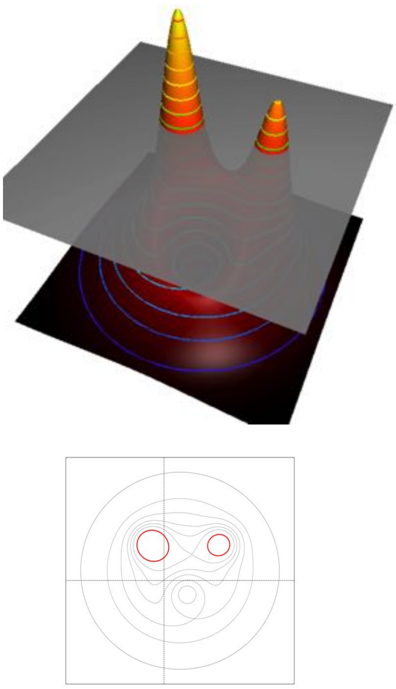
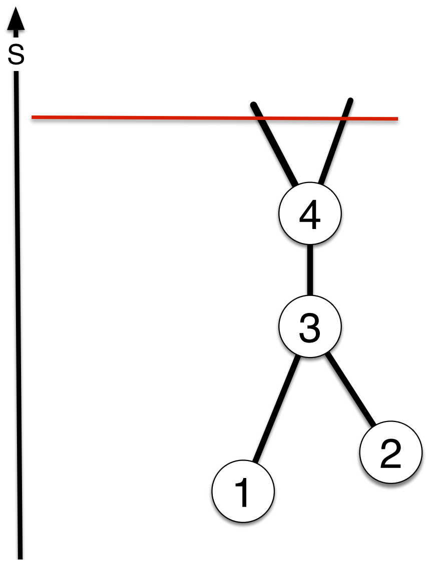




II Topological methods for visualization – Scalar field topology

2 Contour Tree - Illustrative Example





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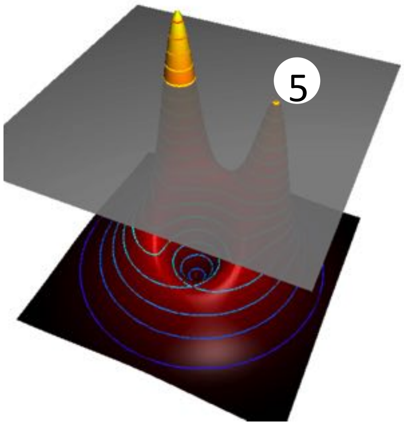
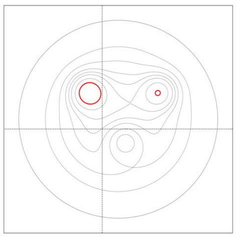
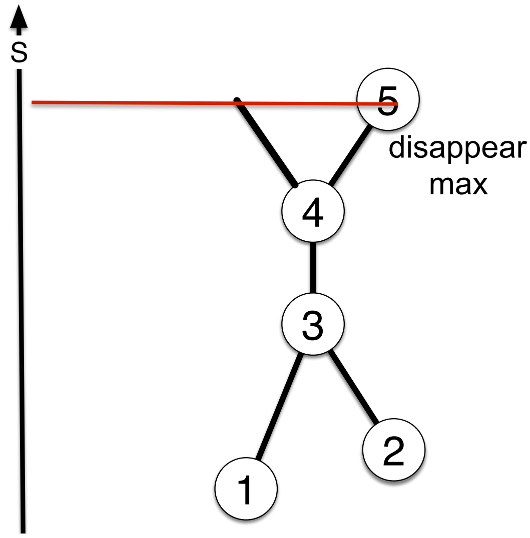
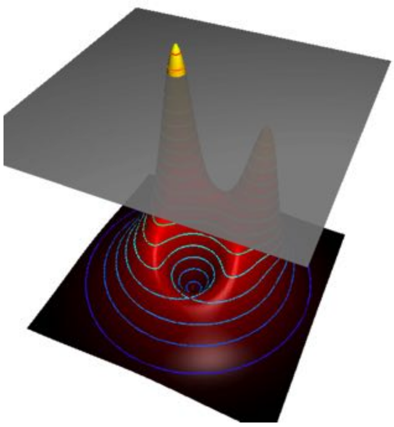


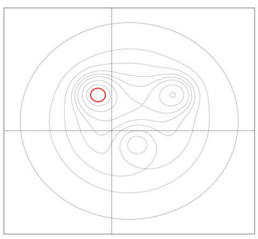


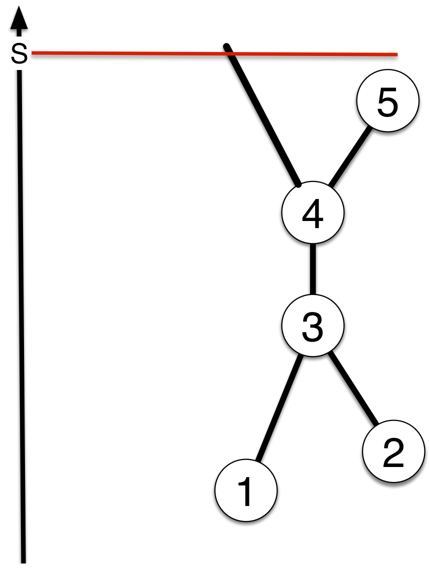
Contour vanishes



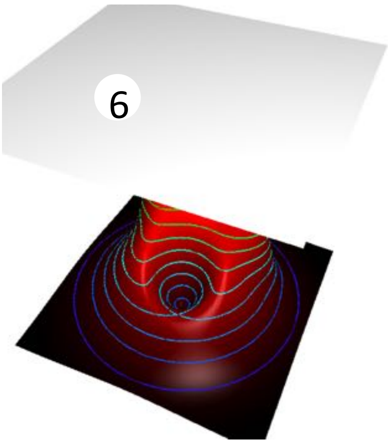
2 Contour Tree - Illustrative Example

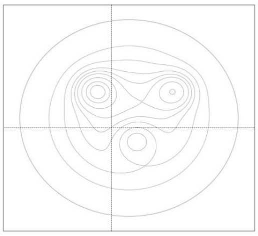


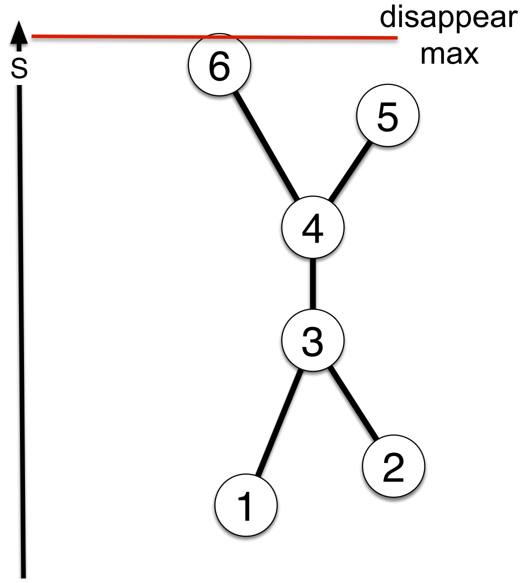




2 Contour Tree - Illustrative Example

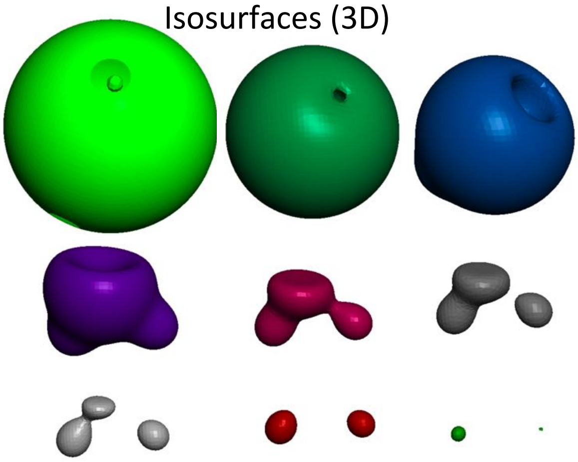




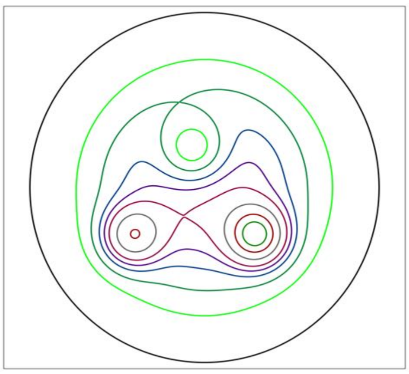


2 Contour Tree - Illustrative Example

Isosurfaces (3D)



Contour lines (2D)



When increasing the isovalue one can observe some **events** where the isosurface/ contour undergoes **characteristic changes: appear, merge, split, disappear** (poke/seal).

→ **Topological analysis keeps track of such changes.**
The locations where these changes appear **are called critical points**, the scalar value **critical value**

2 Contour Tree

- The **contour tree** keeps track of the change of the **number of components** of the contour (isosurface) when changing the scalar value.
  (Introduced by Boyell and Ruston for the evolution of contours on a 2D map).
- The contour tree does **not represent all topological changes**, e.g., the change of topology of a specific contour (from disc to torus).
- Recorded events are
  - Component appears, disappears, components merge and split.

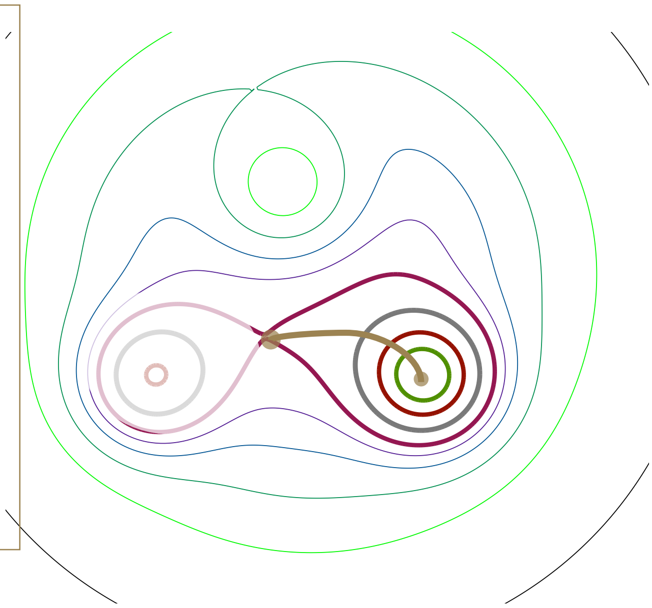
Note

There are some isosurface changes related to the domain boundary, in our discussion we will neglect boundary cases.

2 Contour Tree

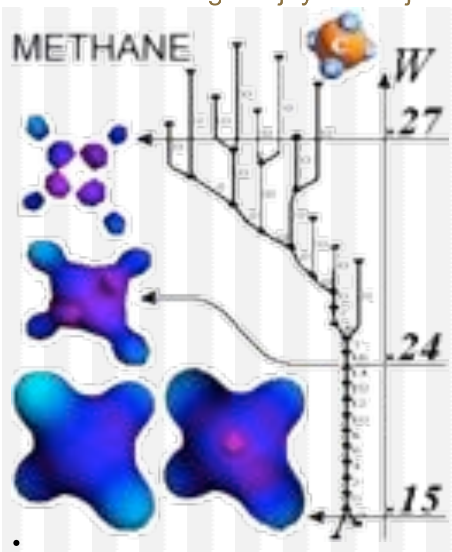
Contour class

- A contour class is an **equivalence class of contours** that can be **smoothly transformed** into each other
- A contour class is **created** or **destroyed** in critical points
- Contour classes are represented by **arcs in the contour tree**



2 Contour Tree

Image: Vijay Natarajan



Definition

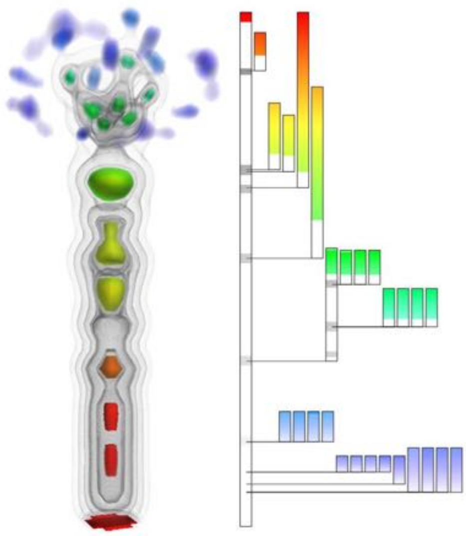
The **contour tree** is a **graph (V,E).**
V are **nodes** and *E* **arcs**

- **The set *V*** contains a node for each **critical point**
- **The set *E*** contains a arc for each **contour class**

Reference for algorithmic solutions, e.g
Computing contour trees in all dimensions, Hamish Carr and Jack Snoeyink and Ulrike Axen, SODA '00: ACM-SIAM symposium on Discrete algorithms, 2000

2 Contour Tree – Application

Topology-controlled Volume Rendering
[Fuel data set, Weber et al. TVCG 2006]



Determine interesting isovalues

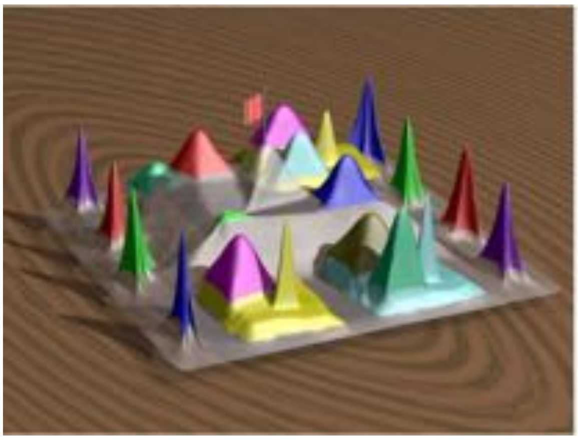
- Guided exploration of a data set
- Represent each contour class

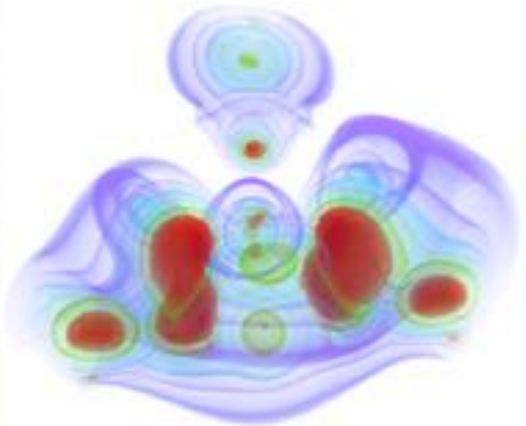
Generate transfer functions for volume rendering

2 Contour Tree – Application

Topological landscape

- Non-overlapping presentation of the topological structure as a topological landscape profile (considers only first order topology)
- Is also applicable to high dimensional data





Visualizing nD Point Clouds as Topological Landscape Profiles to Guide Local Data Analysis, Oesterling et al., TVCG 2013

2 Basic Concept
Critical Points

3 Basic Concept - Critical Points

- **So far** critical point have been defined with respect to topological changes of level sets **(global structures)**.
- However, while topology is a **global concept**, critical points can be defined **locally.**

3 Basic Concept - Critical Points

Critical points – 1D

Let $f : I = [a,b] \rightarrow \mathbb{R}$

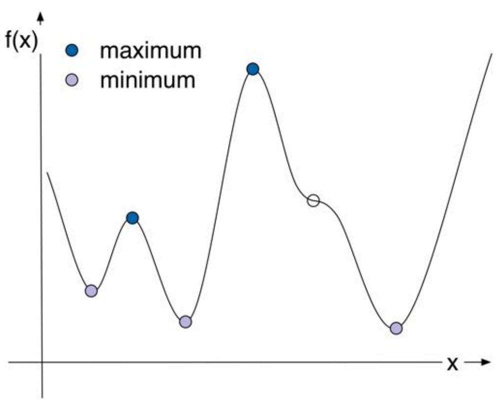
be a differentiable scalar function.

Definition:

A point $x_0 \in I$ is called **critical point** of f if

$$f'(x_0) = 0$$

The scalar value $s_0 = f(x_0)$ is called **critical value**



Critical points can be **classified** with respect to the second derivative in the point.

$$f'' \begin{cases} < 0 & \text{max} \\ > 0 & \text{min} \\ = 0 & \text{degenerate} \end{cases}$$

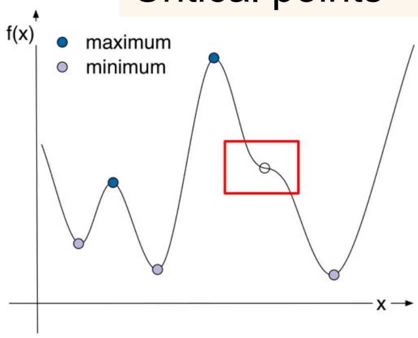
3 Basic Concept - Critical Points

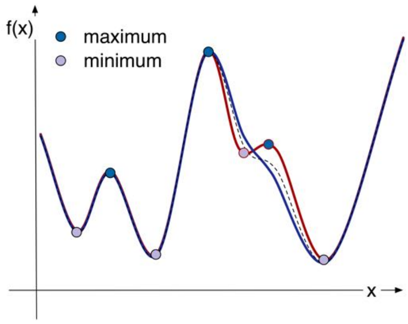
Critical points – 1D

- **Degenerate critical point**s are **not stable** under small perturbations.
- Non-degenerate critical points are stable under small perturbations.

- The function *f* is called **Morse function** if none of its critical points are degenerated.

Note
- Many algorithms assume that there are no degenerate critical points in the data
- However this is not a practical problem





3 Basic Concept - Critical Points

Critical points – 2D

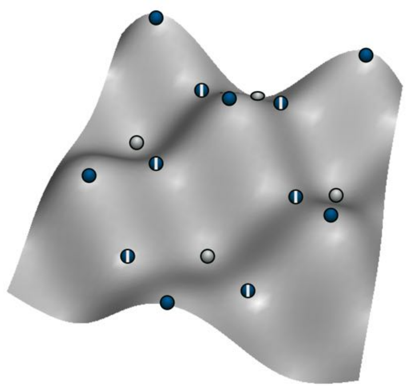
Let $S : \mathbb{R}^2 \supset D \rightarrow \mathbb{R}$

be a differentiable scalar function.

Definition:

A point $\mathbf{x}_0 \in D$ is called **critical point** of S if

$$\nabla S = \begin{pmatrix} \frac{\partial S}{\partial x} & \frac{\partial S}{\partial y} \end{pmatrix} = 0$$



- **Classified** with respect to the second derivative **the Hessian** $\nabla^2 S(\mathbf{x}_0)$
- Consider the **sign of its eigenvalues**

$$\nabla^2 S = \begin{pmatrix} \frac{\partial^2 S}{\partial x^2} & \frac{\partial^2 S}{\partial x \partial y} \\ \frac{\partial^2 S}{\partial y \partial x} & \frac{\partial^2 S}{\partial y^2} \end{pmatrix}$$

3 Basic Concept - Critical Points

Critical points – nD

Let $S : \mathbb{R}^n \supset D \to \mathbb{R}$

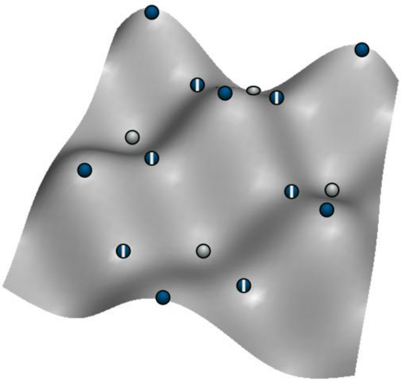
be a differentiable scalar function.

Definition:

A point $\mathbf{x}_0 \in D$ is called **critical point** of S if

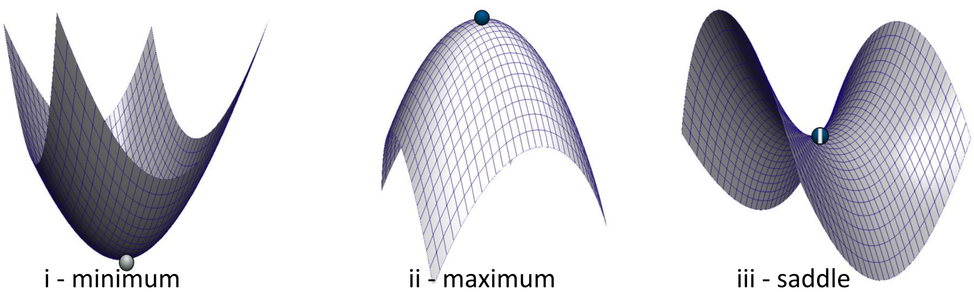
$$\nabla S = \left(\frac{\partial S}{\partial x_1}, \ldots, \frac{\partial S}{\partial x_n} \right) = 0$$

- **Classified** with respect to the second derivative **the Hessian** $\nabla^2 S(\mathbf{x}_0)$
- Consider the **sign of its eigenvalues**



$$\nabla^2 S = \begin{pmatrix} \frac{\partial^2 S}{\partial x_1^2} & \cdots & \frac{\partial^2 S}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 S}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 S}{\partial x_n^2} \end{pmatrix}$$

3 Basic Concept - Critical Points



i - minimum ii - maximum iii - saddle

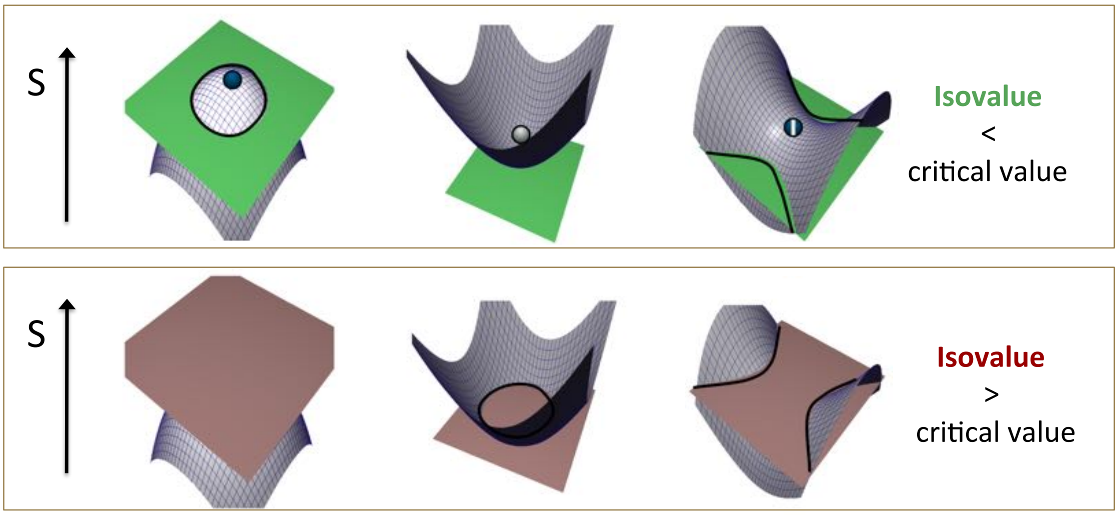
Classification of critical points – using an index (number of negative coefficients)

Index	n=1	n=2	n=3
i=0	x^2	x^2+y^2	$x^2+y^2+z^2$
i=1	$-x^2$	$-x^2+y^2$	$-x^2+y^2+z^2$
i=2		$-x^2-y^2$	$-x^2-y^2+z^2$
i=3			$-x^2-y^2-z^2$

3 Basic Concept - Critical Points

Change of isosurface when passing a critical value – 2D field

Scalar field S represented as height field,

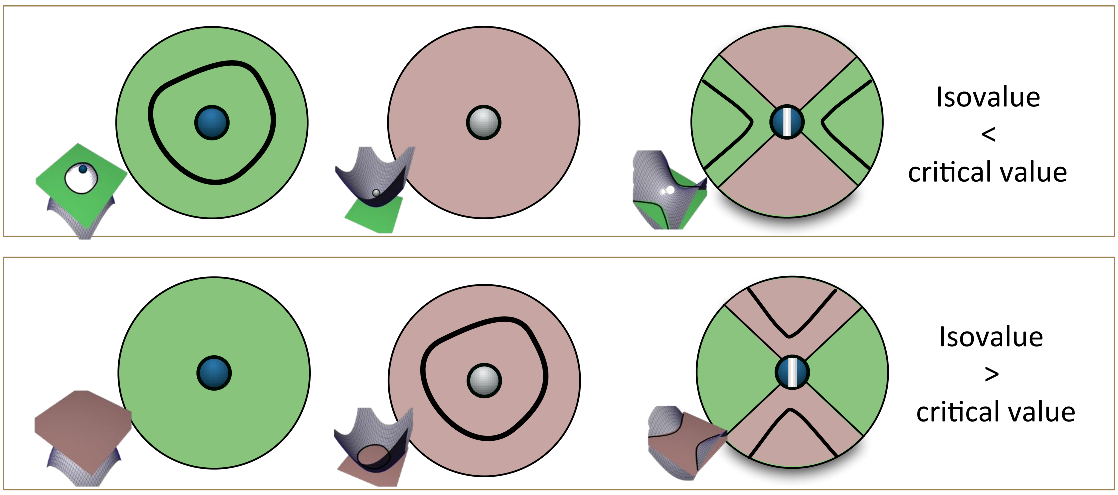


Contour Maximum disappears Minimum appears Saddle changes connectivity

3 Basic Concept - Critical Points

Change of isosurface when passing a critical value – 2D field

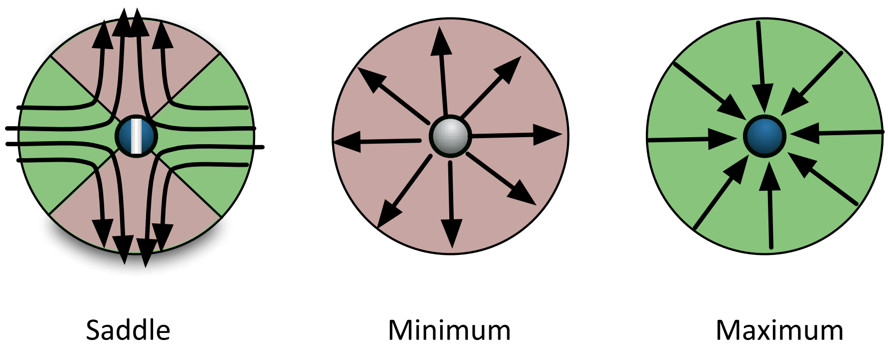
Projection in domain, color represents scalar value with respect to critical value



Contour Maximum disappears Minimum appears Saddle changes connectivity

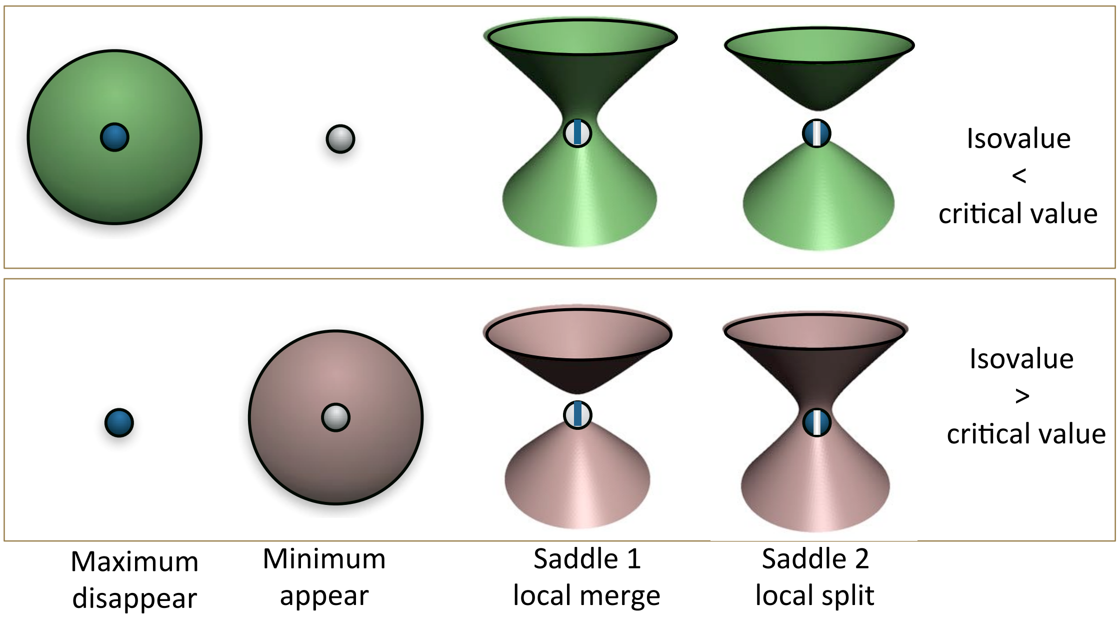
3 Basic Concept - Critical Points

Gradient lines -2D field.



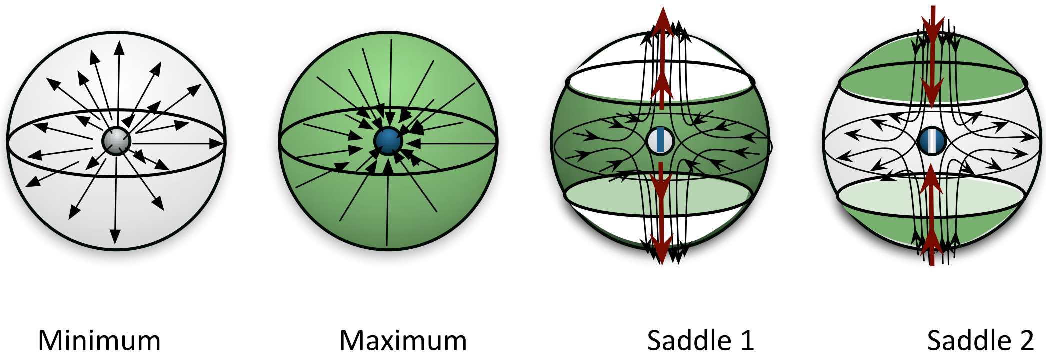
3 Basic Concept - Critical Points

Change of isosurface when passing a critical value – 3D field.



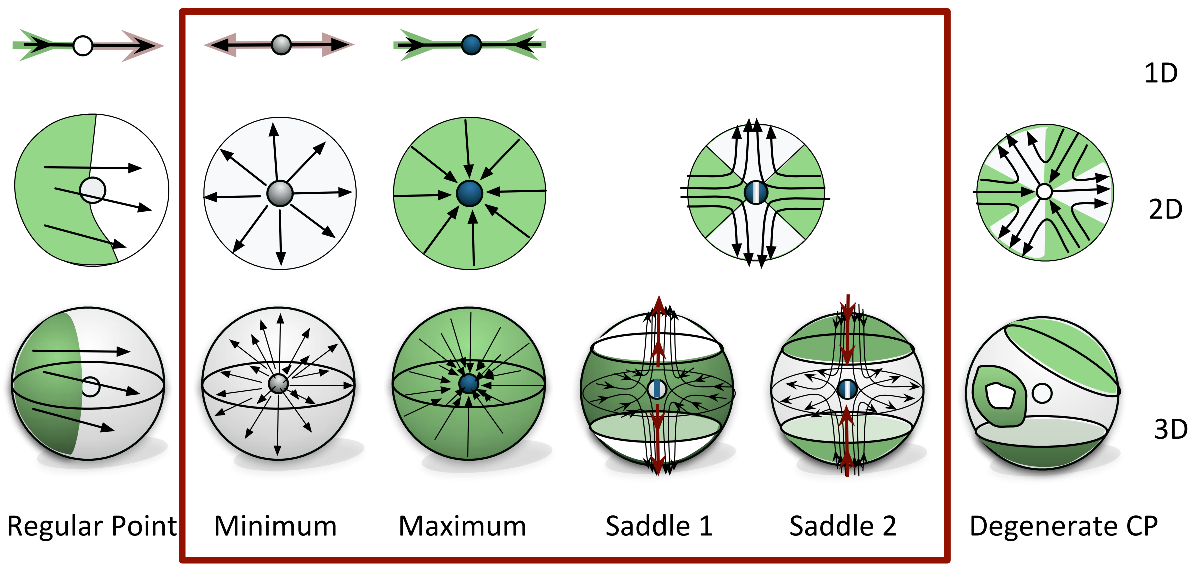
3 Basic Concept - Critical Points

Gradient lines -3D field.



3 Basic Concept - Critical Points

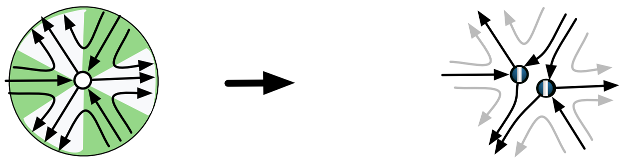
Point classifications – overview



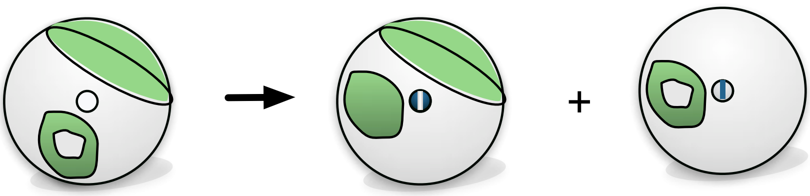
3 Basic Concept - Critical Points

Higher order degenerate points are not stable

E.G Splitting saddles:



2D



3D

3 Morse Smale complex

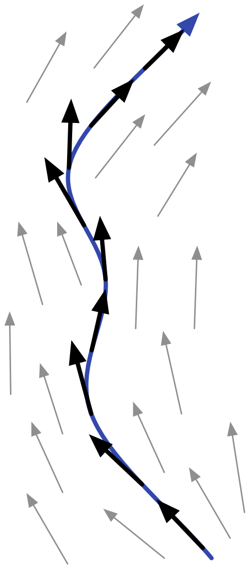
3 Morse Smale complex

Consider gradient vector field

- Gradient in critical points is zero
- Integral lines / streamlines maximal open curve tangential to the gradient

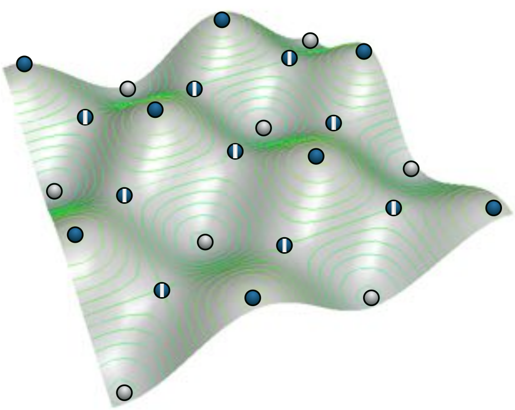
Properties of integral lines

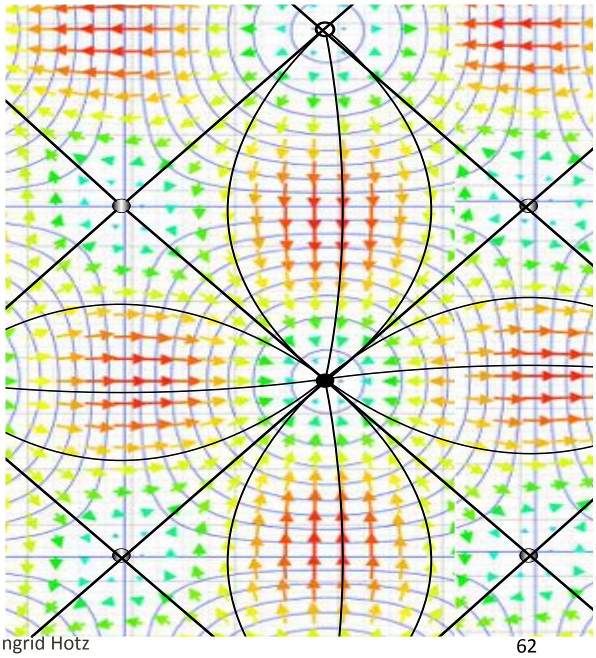
- They cover all regular points in domain
- They 'start' and 'end' in critical points
- They are monotonic



3 Morse Smale complex

Gradient vector field



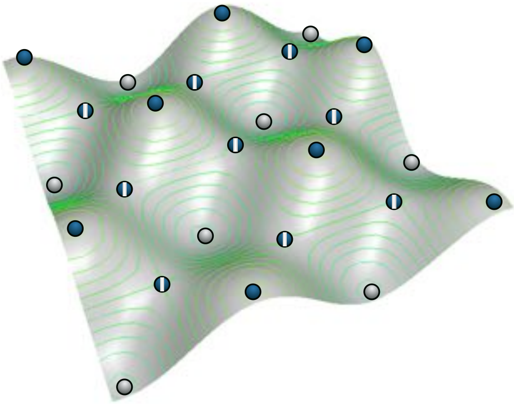


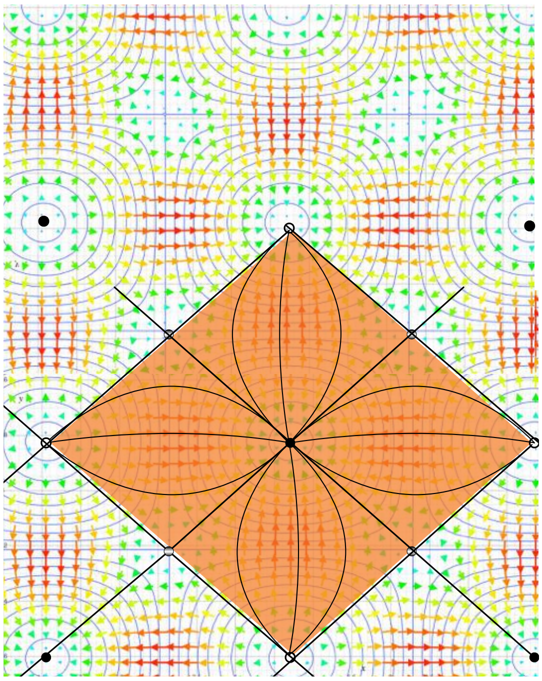
3 Morse Smale complex

Gradient vector field

- Critical point and its descending manifold

Descending manifold:
Set of points that converge toward the critical point



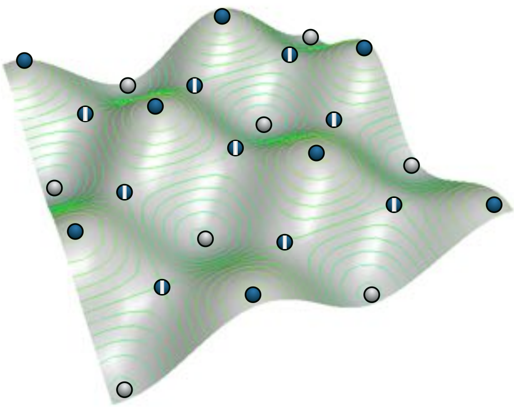


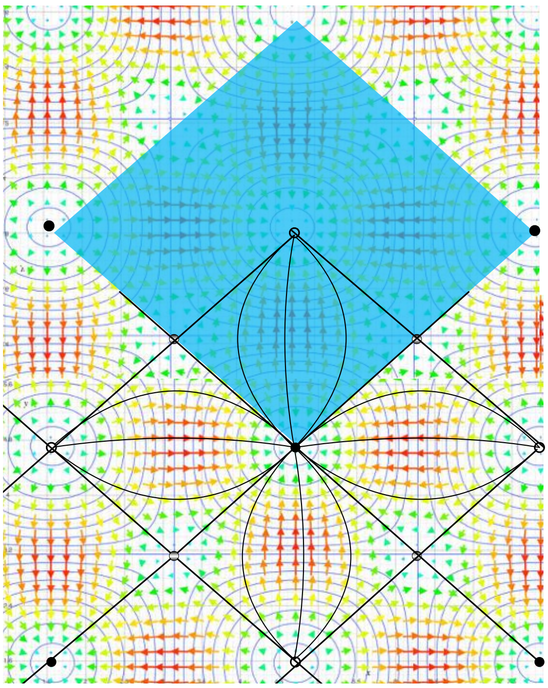
3 Morse Smale complex

Gradient vector field

- Critical point and its ascending manifold

Ascending manifold:
Set of points that emerge from the critical point

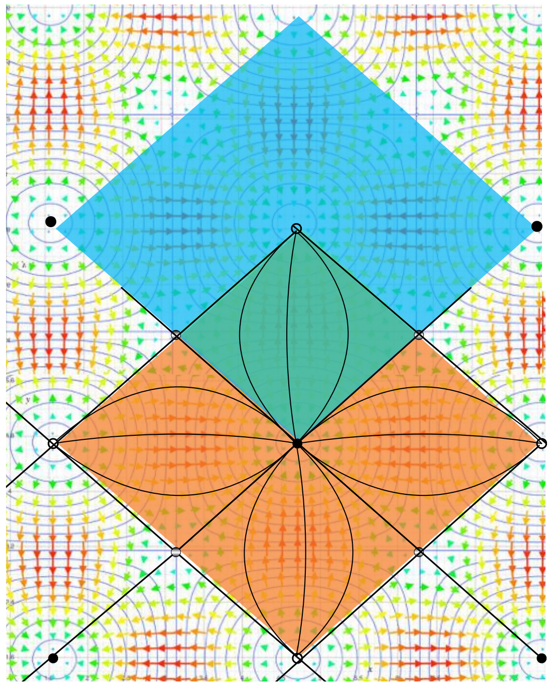


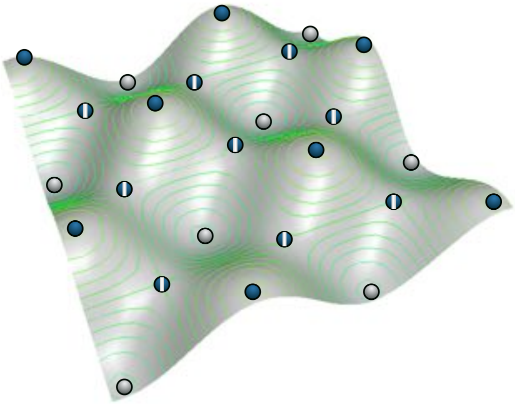


3 Morse Smale complex

Gradient vector field

- Critical points and its descending / ascending manifold





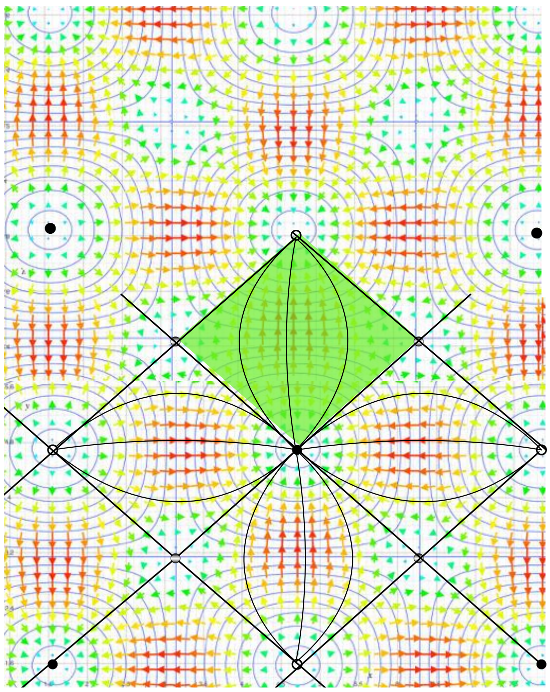
3 Morse Smale complex

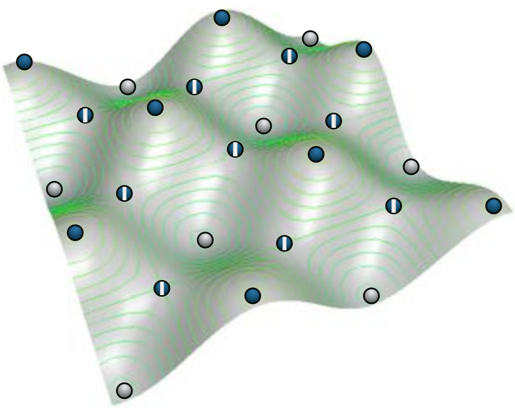
Gradient vector field

- One Morse cell

Morse cell

- Set of points that emerge from one critical point converging two a second critical point





3 Morse Smale complex

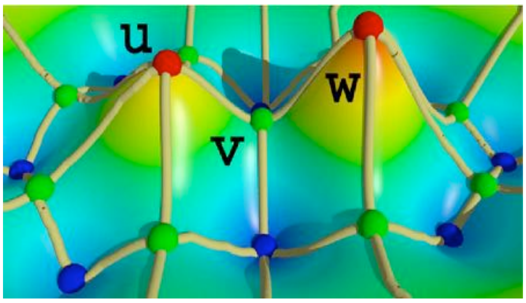
Morse Smale complex

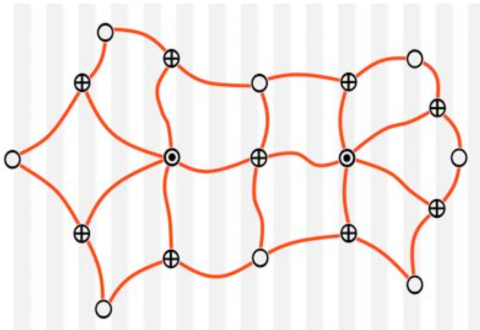
- Decomposition / segmentation of the domain into monotonic quadrangular regions by connecting critical points with lines of steepest descent (separatrices)

For all integral curves in one cell

- Joined origin = minimum
- Joined destination = maximum

→ Equivalence classes of integral curves



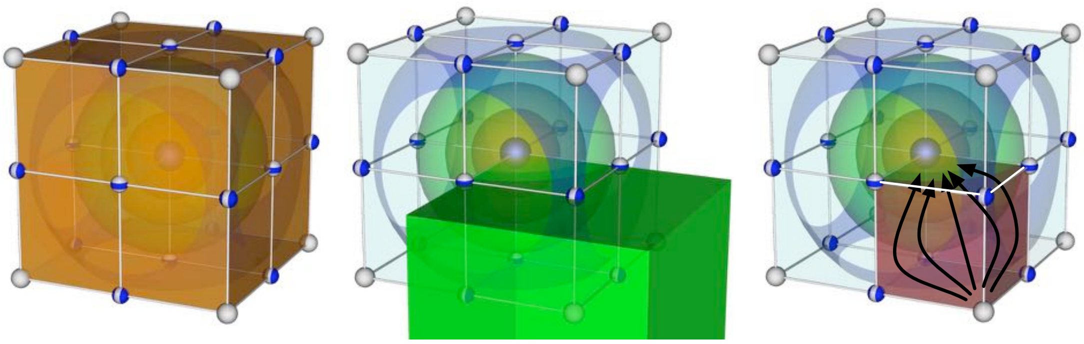


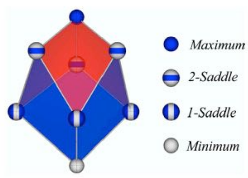
Images: V. Natarajan

3 Morse Smale complex

Morse cells in 3D

Ascending manifold | Descending manifold | Morse cell





Images: V. Natarajan

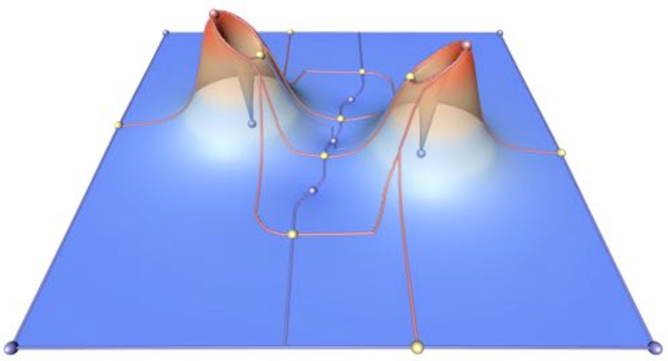
4 Extremal structures

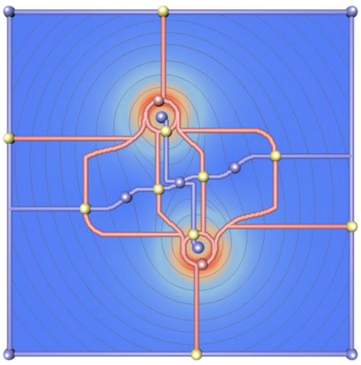
4 Extremal structures

Example – vortex extraction

- **Vortex core,** acceleration minima
- **Vortex region** corresponding basin

→ There is no appropriate **iso-value** to cover the features as contours
→ Features are **extremal structures** of acceleration field





Two co-rotating Oseen Vortices, images: Jens Kasten
Height and color – acceleration magnitude

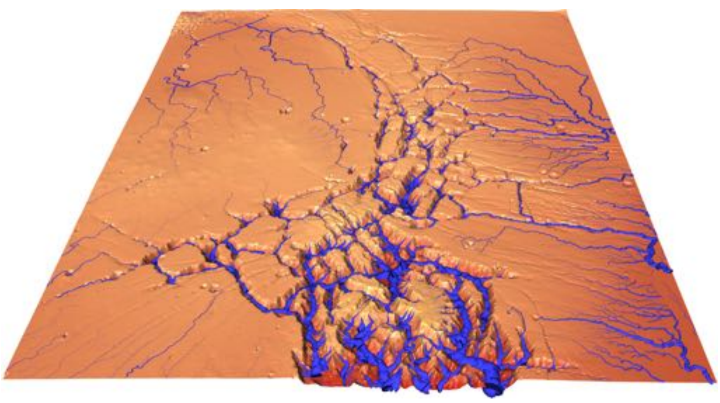
4 Extremal structures

Extremal structures

- a simplified substructure of the Morse-Smale complex that encodes how neighboring extrema are connected via “ridge”- or “valley”-like saddle points.

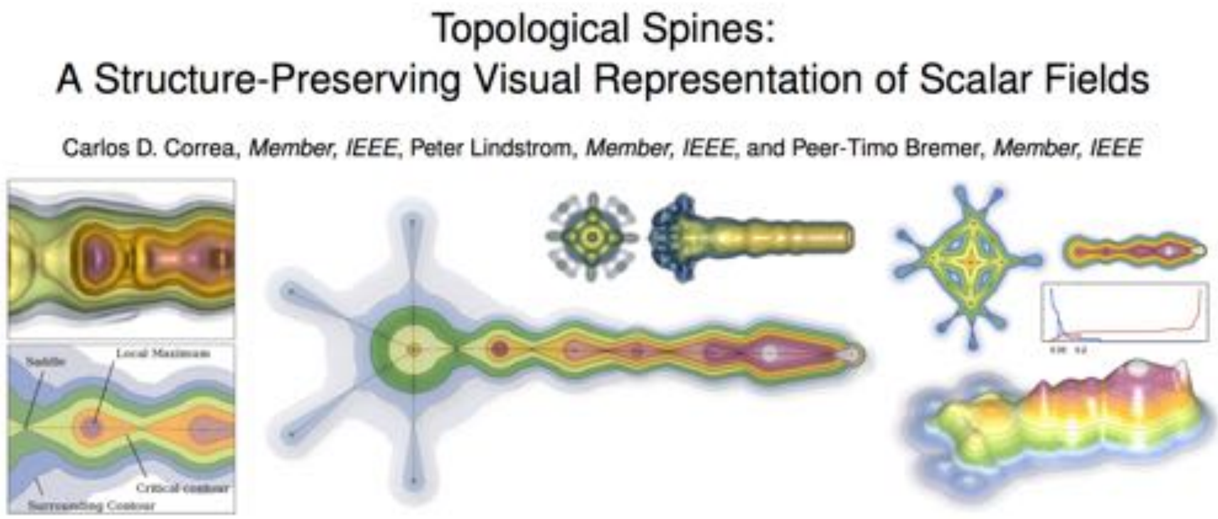
4 Extremal structures

Example - Extraction of ridges and valleys as surface features



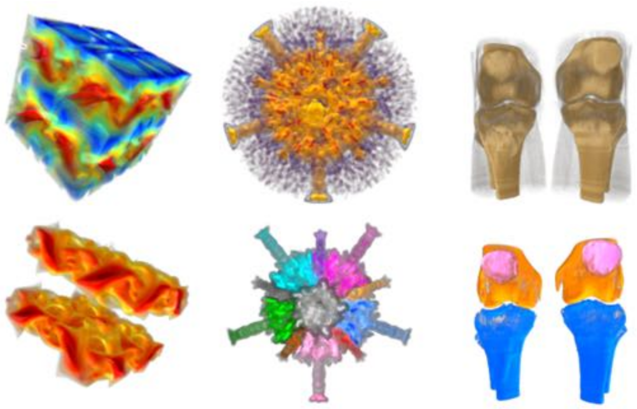
Data: Height field of the Martian surface form imaging
Image: Gunther, ZIB

4 Extremal structures


Topological Spines:
A Structure-Preserving Visual Representation of Scalar Fields

Carlos D. Correa, *Member, IEEE*, Peter Lindstrom, *Member, IEEE*, and Peer-Timo Bremer, *Member, IEEE*

4 Extremal structures



Topologically defined symmetries

- Left: temperature distribution in a vortex flow simulation,
- Center: cryo-electron microscopy image of a virus,
- Right: CT scan image of a pair of knees.

Symmetry-aware transfer function (bottom)

- Left: identifying similar subtrees of the **contour tree**,
- Center: comparing **distance**s between extrema **using extremum graph**,
- Right**: clustering contours** in a high dimensional **shape descriptor space**.

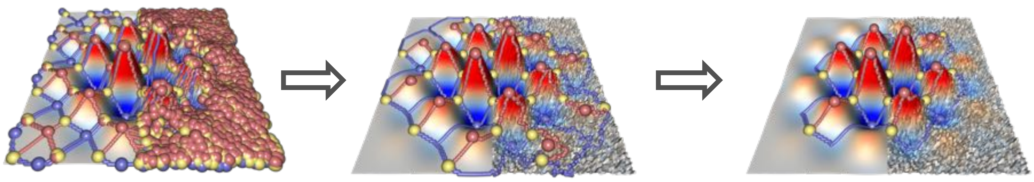
Symmetry in scalar field topology. Thomas and Natarajan.
TVCG *(Vis 2011)*, 17(12), 2011, 2035-2044.

5 Simplification

5 Simplification

In real data sets the feature density is often very high

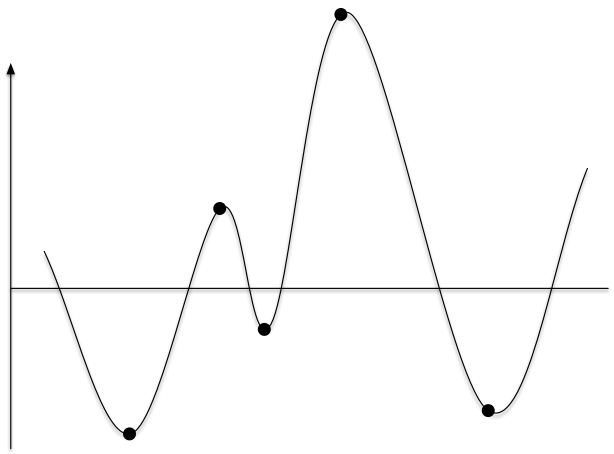
- Can we distinguish real feature from spurious noise related features?
- Is there a way to measure the relevance of features even beyond noise.



Example: Noisy gradient vector field
Images: Reininghaus, ZIB

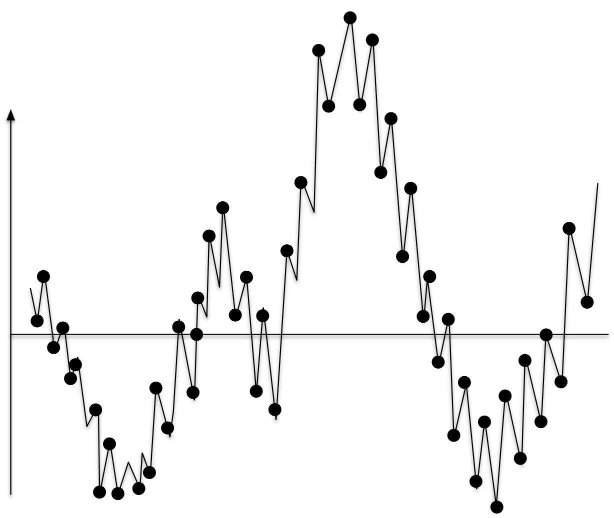
5 Simplification

Question: What are relevant features?



5 Simplification

Question: What are relevant features?



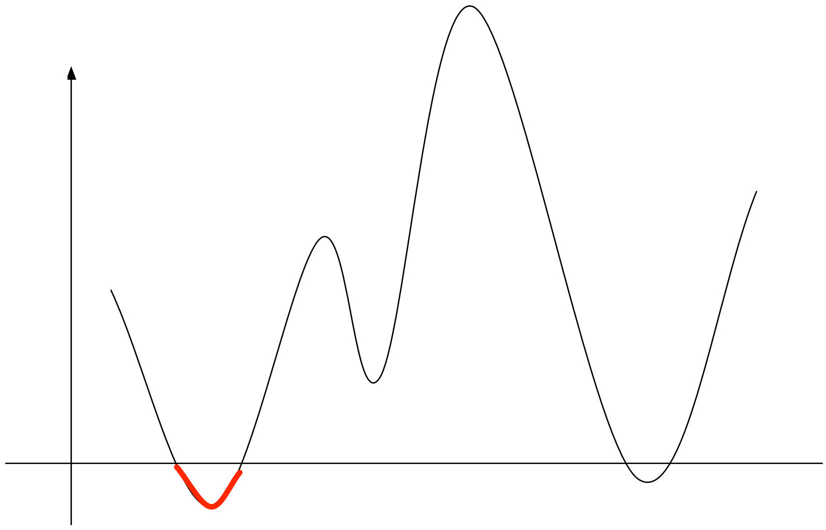
Relevance of critical points cannot be locally decided

5 Simplification

Topological persistence [Edelsbrunner 2002]

- Idea: consider "lifetime" of a feature

Upward sweep
- birth



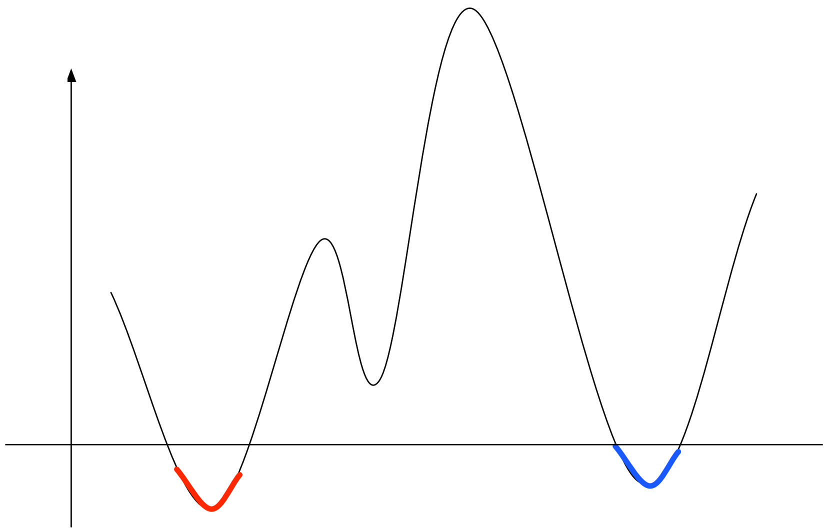
II Topological methods for visualization – Scalar field topology

5 Simplification

Topological persistence

- Idea: consider "lifetime" of a feature

Upward sweep
- birth
- birth



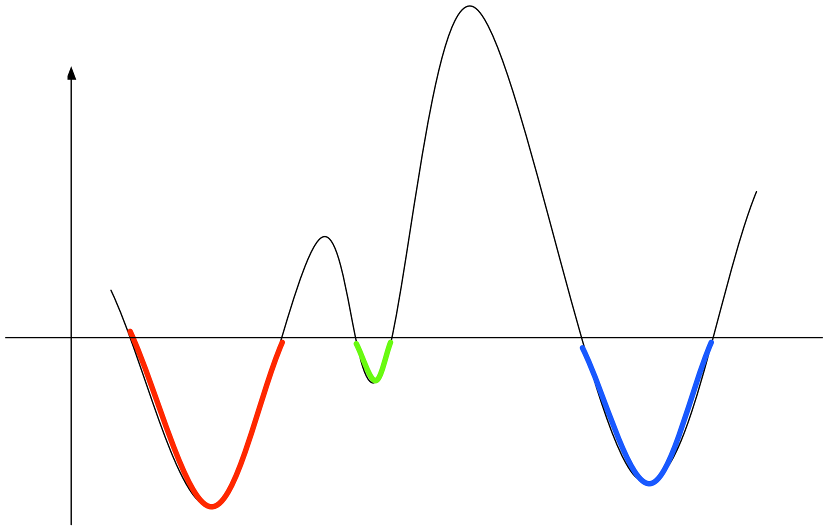
5 Simplification

Topological persistence

- Idea: consider "lifetime" of a feature

Upward sweep

- birth
- birth
- birth



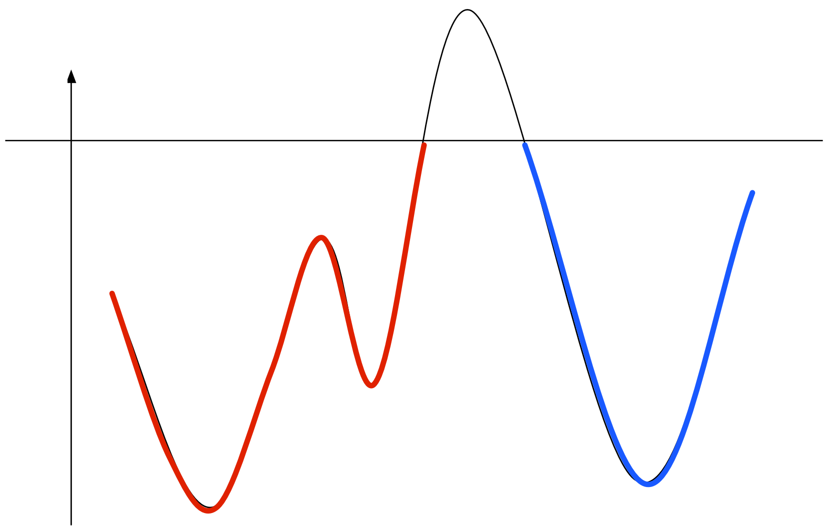
5 Simplification

Topological persistence

- Idea: consider "lifetime" of a feature

Upward sweep

- birth
- birth
- birth
- death

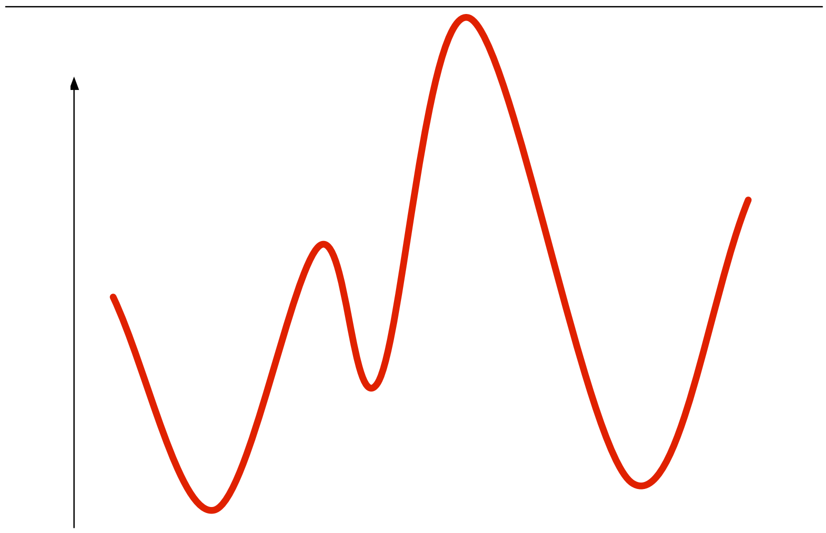


5 Simplification

Topological persistence

- Idea: consider "lifetime" of a feature

Upward sweep
- birth
- birth
- birth
- death
- death



5 Simplification

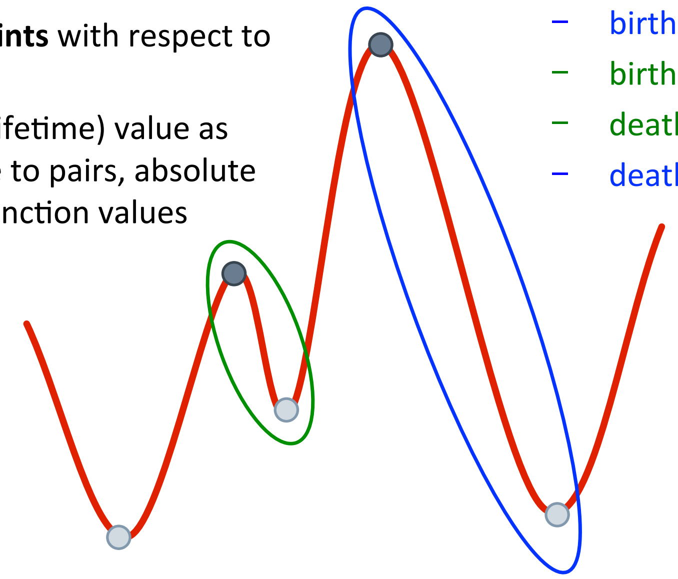
Topological persistence

- Idea: consider "**lifetime**" of a feature

→ **pairing of critical points** with respect to birth and death

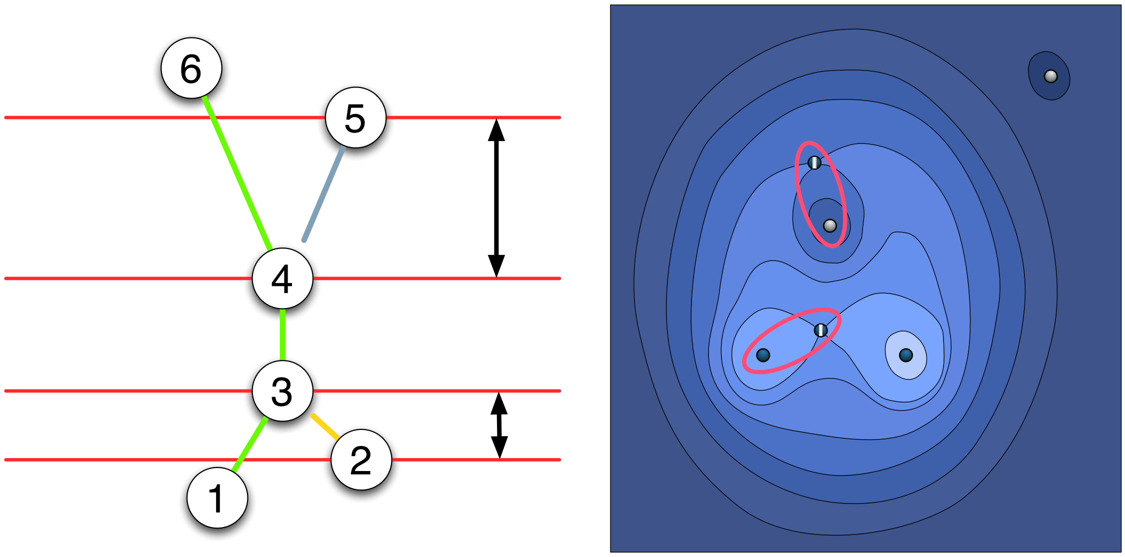
→ Assign persistence (lifetime) value as importance measure to pairs, absolute difference of their function values

Upward sweep
- birth
- birth
- birth
- death
- death



5 Simplification

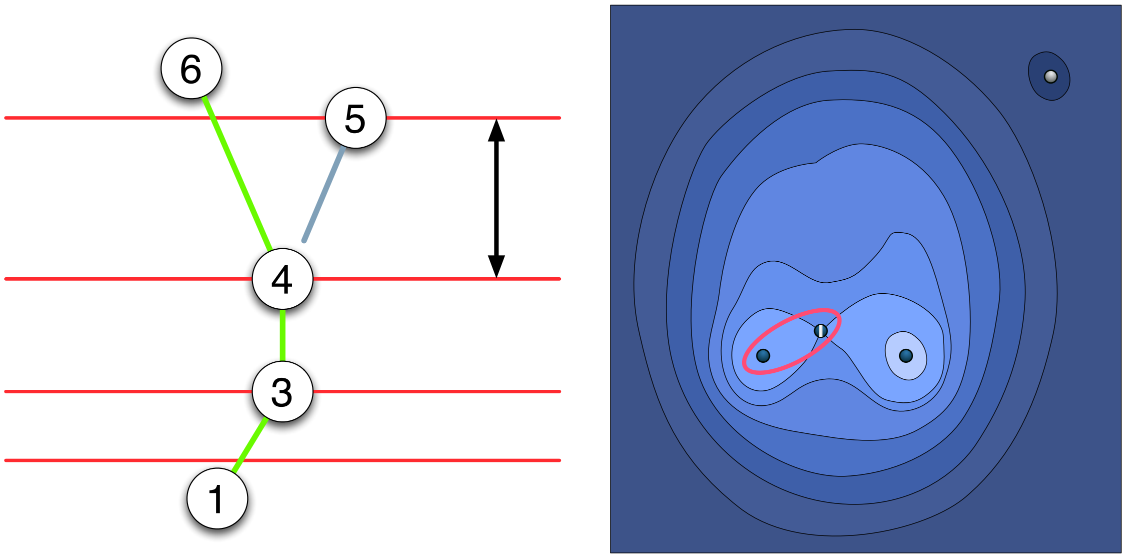
Contour tree simplification



Order the pairs of critical points based on their persistence

5 Simplification

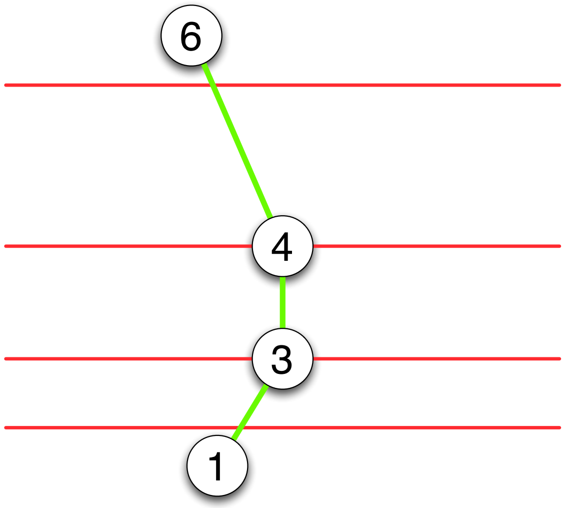
Contour tree simplification

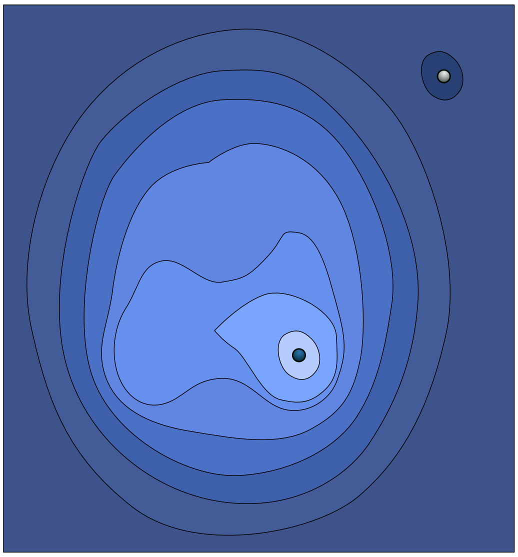


Order the pairs of critical points based on their persistence

5 Simplification

Contour tree simplification

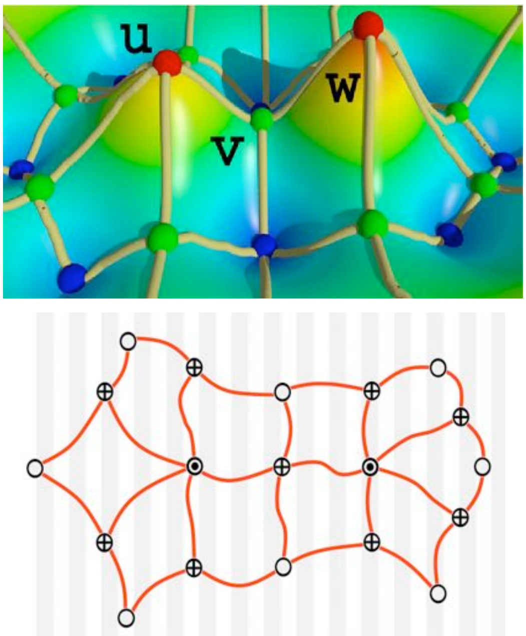


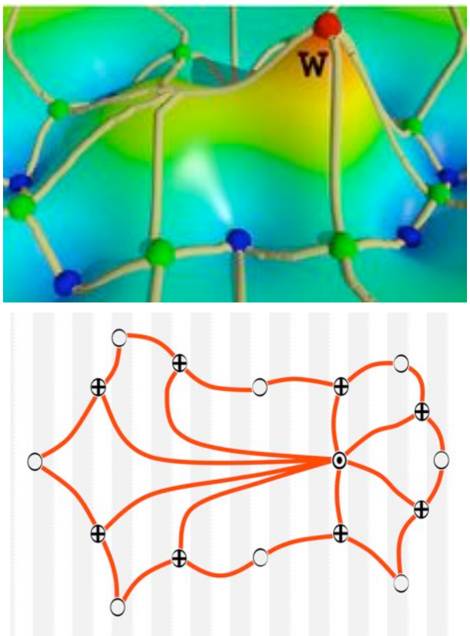


Order the pairs of critical points based on their persistence

5 Simplification

Cancellation for saddle extremum pairs

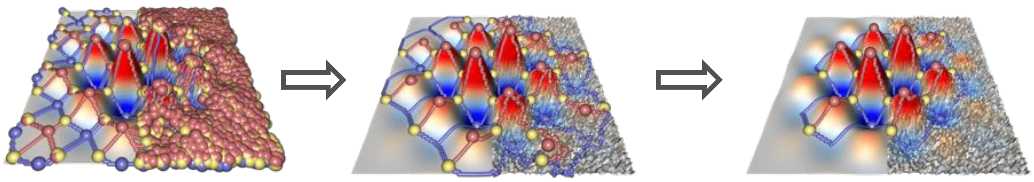




Images: V. Natarajan

5 Simplification

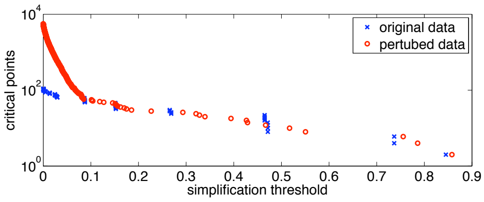
Example: Noisy gradient vector field



Topological Features of the original field

Features associated with noise are removed

Simplified version reflects dominant structures

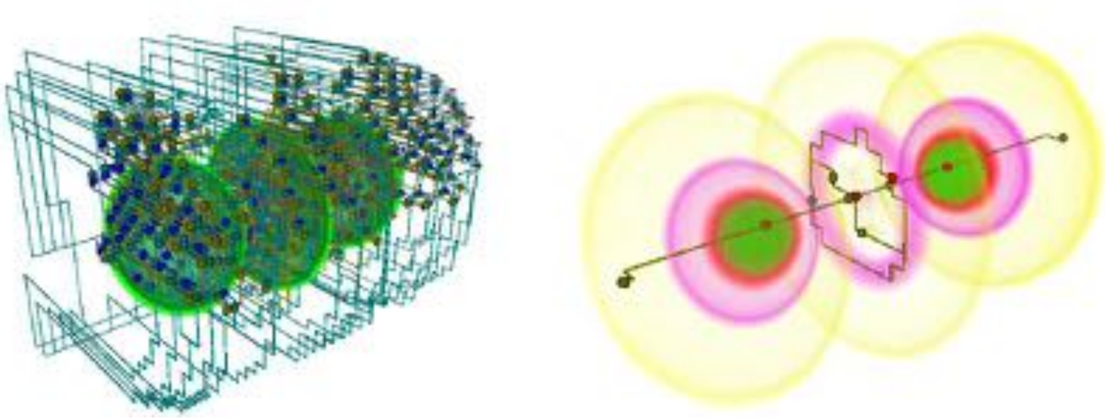


Number of critical points in dependence from hierarchy level.

The transition from noisy to real features is clearly visible as change of slope

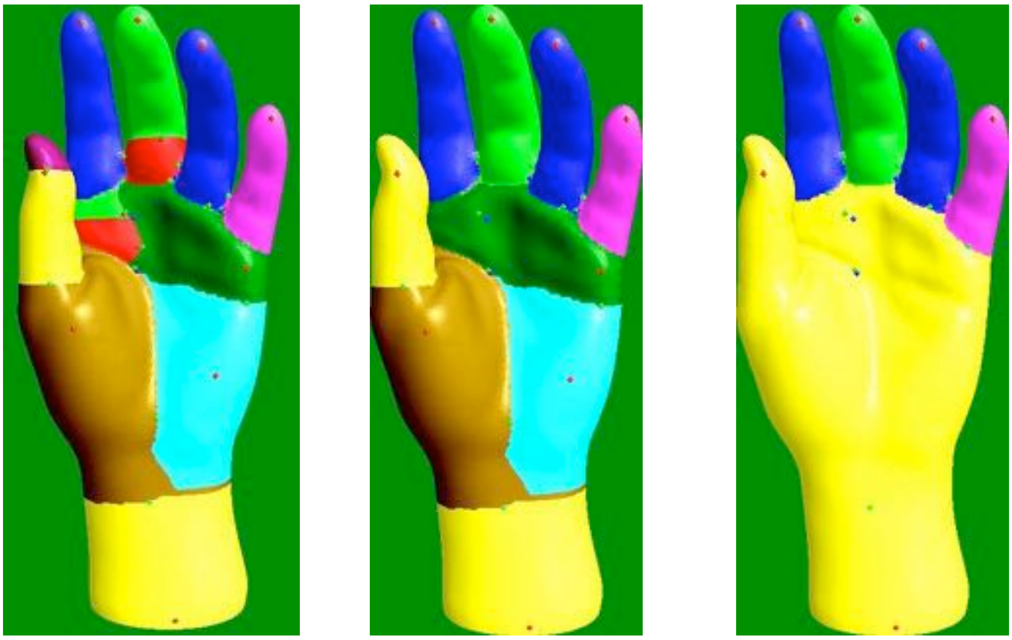
Images: Reininghaus

4.1.9 Morse Smale Complex - Cancellation



Images: V. Natarajan

4.1.9 Morse Smale Complex – Descending Manifold



Images: V. Natarajan

Overview

I. Introduction

II. Scalar field topology

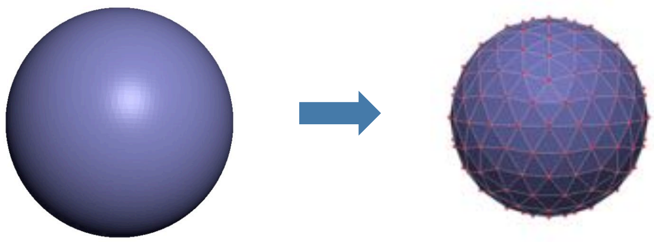
1. Contour tree
2. Critical points
3. Morse Smale complex
4. Extremal structures
5. Simplification
6. **From analytical concepts to discrete realizations**
7. **Examples from flow visualization**

III. Vector field topology

IV. Tensor field topology

6 From analytical concepts to discrete realizations

6 From analytical concepts to discrete realizations

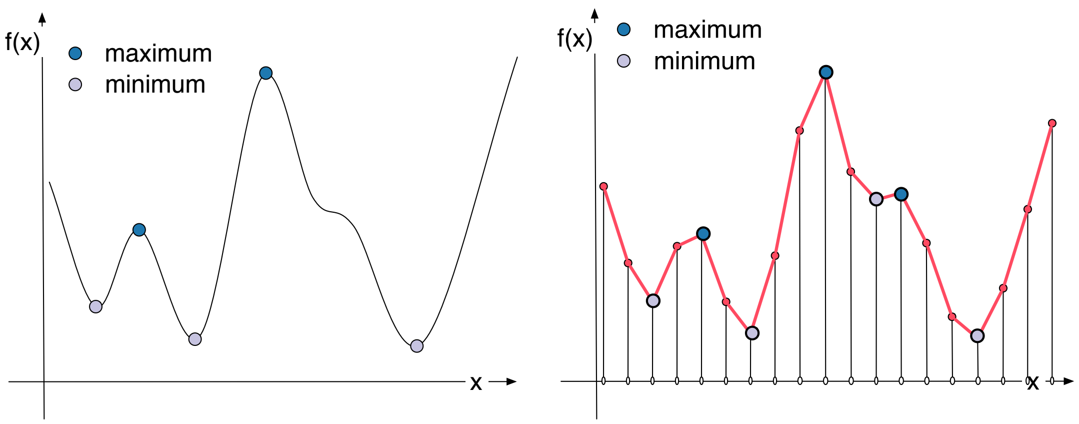


- In general our data sets are **given as samples**.
  - Domain is mostly represented by a mesh (triangulation, tetrahedrization, …)
  - Function values are only available at discrete points
- **Definitions** so far are **based on differentiable functions**

Two options
→ Use **interpolation** to define function everywhere
→ Definitions have to be **generalized to fit into the discrete setting**

6 From analytical concepts to discrete realizations

Simplest solution: consider **piecewise linear functions**

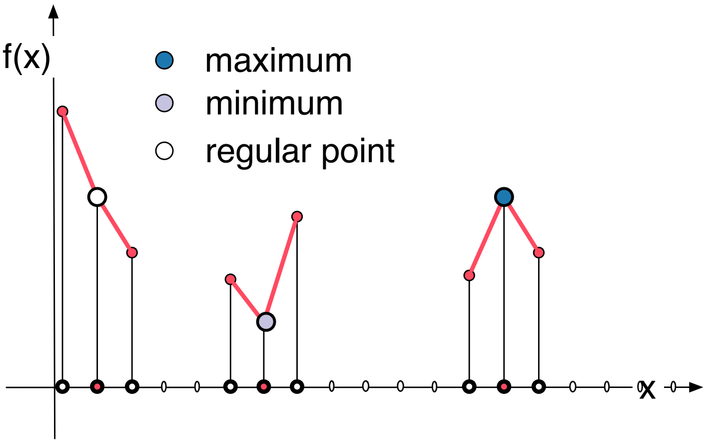


Minima and maxima of piecewise linearly interpolated functions always lay on vertices

6 From analytical concepts to discrete realizations

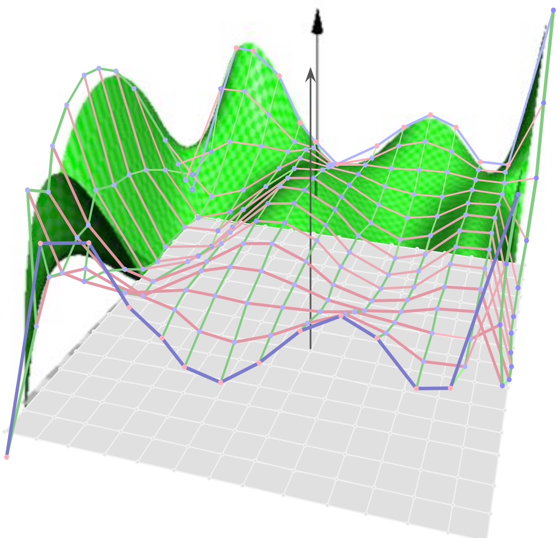
Simplest solution: consider **piecewise linear functions**

- Critical points defined by behavior in neighborhood
- **How is neighborhood defined**?
  Especially critical when moving to higher dimensions (In 1D is easy)



6 From analytical concepts to discrete realizations

Simplest solution: consider **piecewise linear functions**



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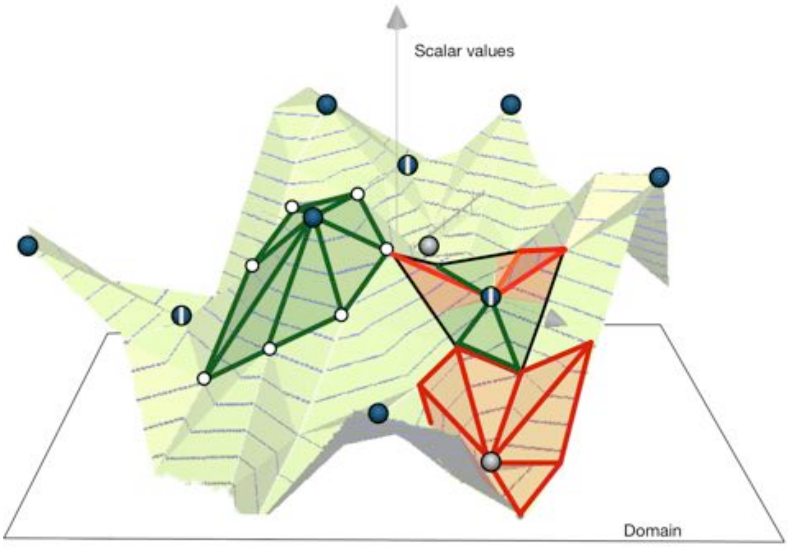
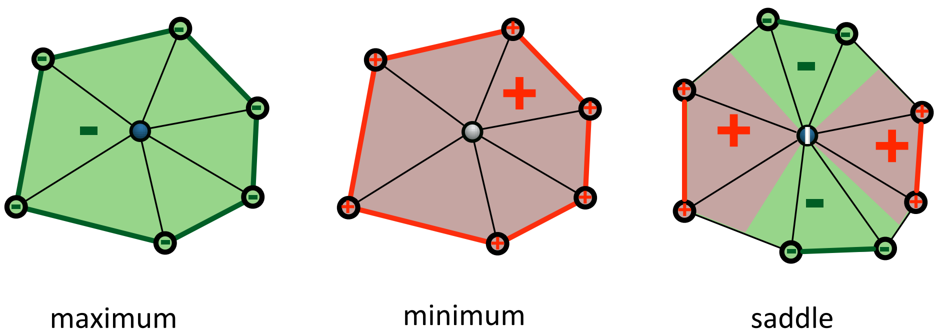
Simplest solution: consider **piecewise linear functions**



6 From analytical concepts to discrete realizations

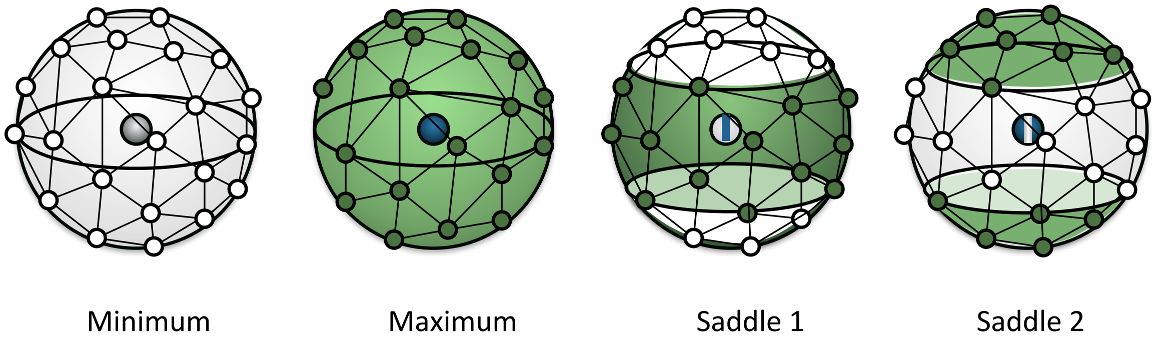


Classification

- **Continuous stetting**: Sign of eigenvalues of Hessian
- **Discrete setting**: Number of connected components (oceans) of positive rep. negative "neighborhood regions".

Reference: *Topology-based Simplification for Feature Extraction from 3D Scalar Fields*, Gyulassy et al. Proceedings of IEEE Visualization,2005

6 From analytical concepts to discrete realizations



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6 From analytical concepts to discrete realizations

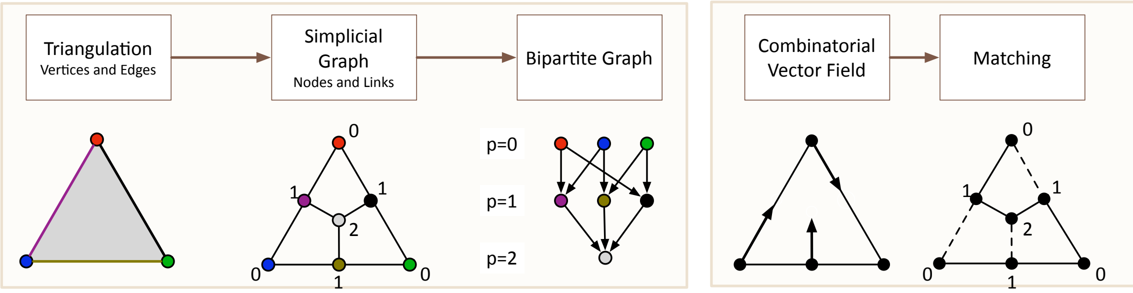
Piecewise linearly interpolated

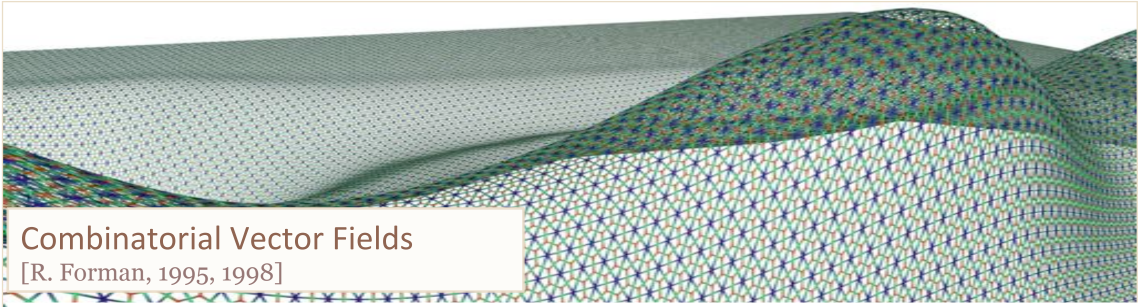
- Theory and algorithms for the extraction can nicely be formulated within the context of **simplicial complexes**
- This setting makes it possible to deal with the continuously defined function *f* using a **combinatorial approach**
- **Simplification, persistence computation boils down to matrix operations**

- However: Critical point detection in higher dimension is getting more and more complex up to bining infeasible

6 From analytical concepts to discrete realizations

Alternative: discrete Morse theory (outlook)





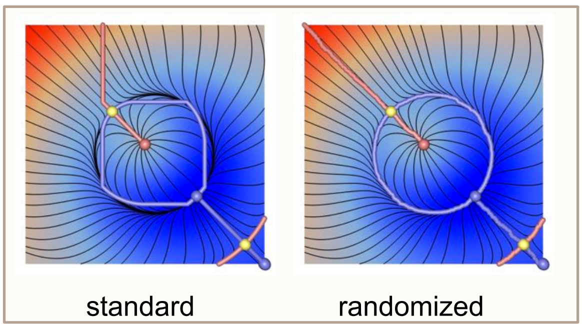
Applications of Forman's Discrete Morse Theory to Topology Visualization and Mesh Compression, Thomas Lewiner and Helio Lopes and Geovan Tavares, TVCG 2004

II Topological methods for visualization – Scalar field topology

6 From analytical concepts to discrete realizations

Remarks:

- **Correctness** of the result can be guaranteed (with respect to the given data)
- For combinatorial approaches the **geometric embedding** of separatrices is not very accurate



Reference: *Combinatorial Gradient Fields for 2D Images with Empirically Convergent Separatrices*, Reininghaus, Günther, Weinkauf, Seidel, Hotz

6 Examples from flow visualization

6 Examples from flow visualization

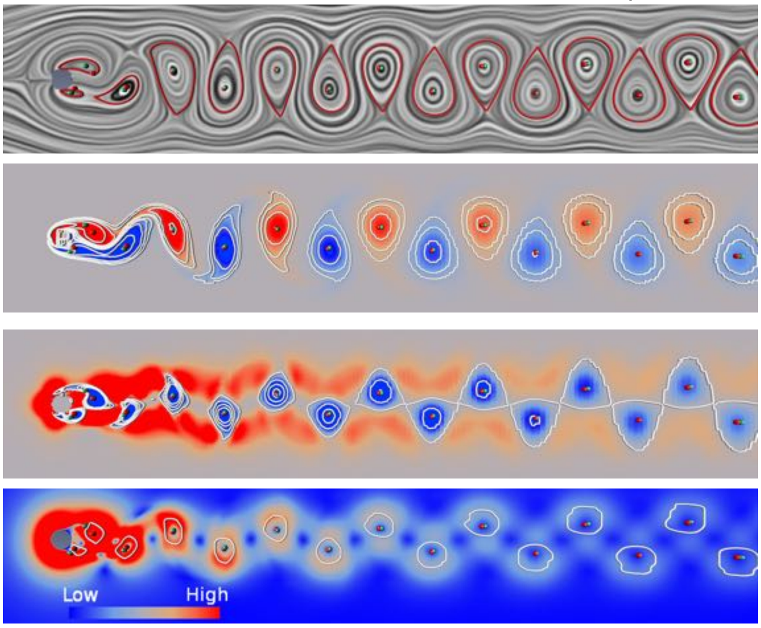
Derived scalar fields as feature identifiers – flow behind a cylinder

Streamlines

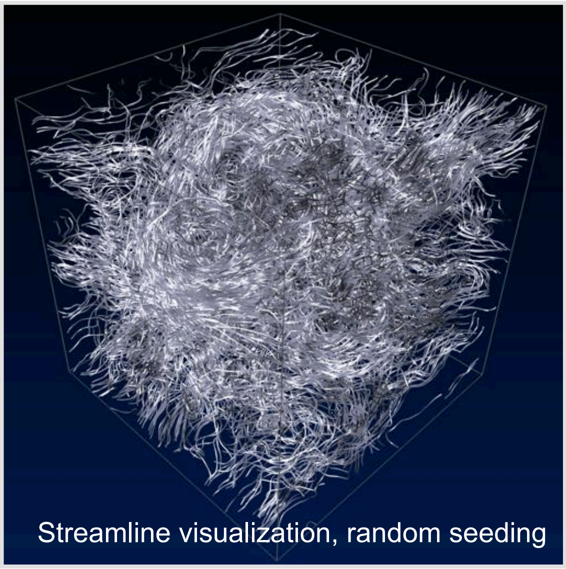
Vorticity

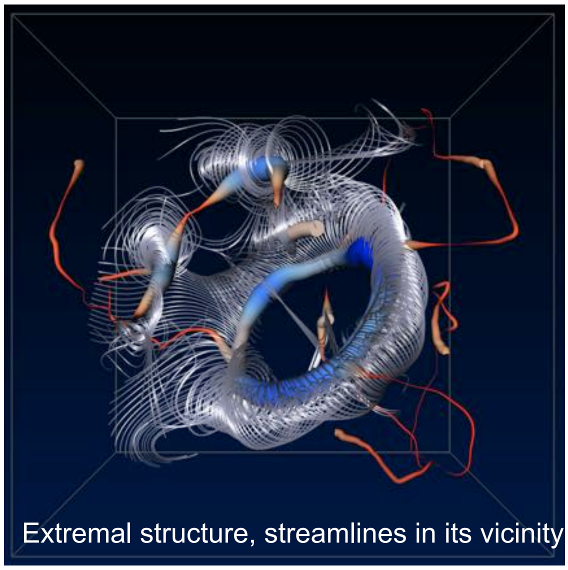
λ2

Acceleration magnitude



6 Examples from flow visualization

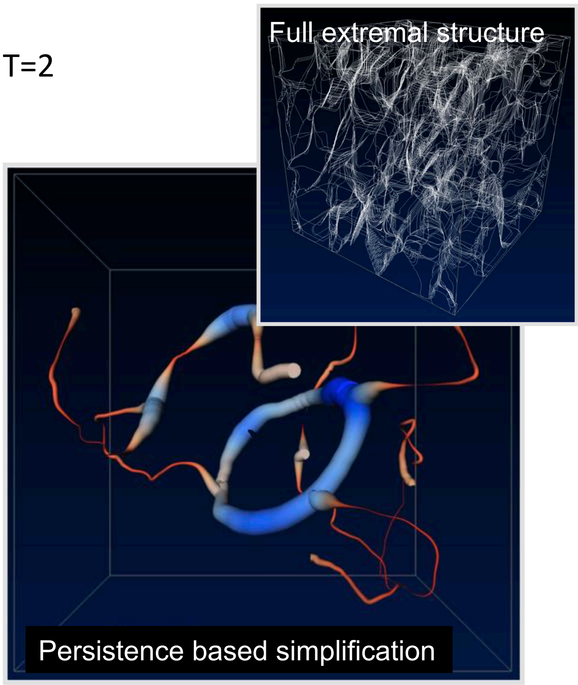




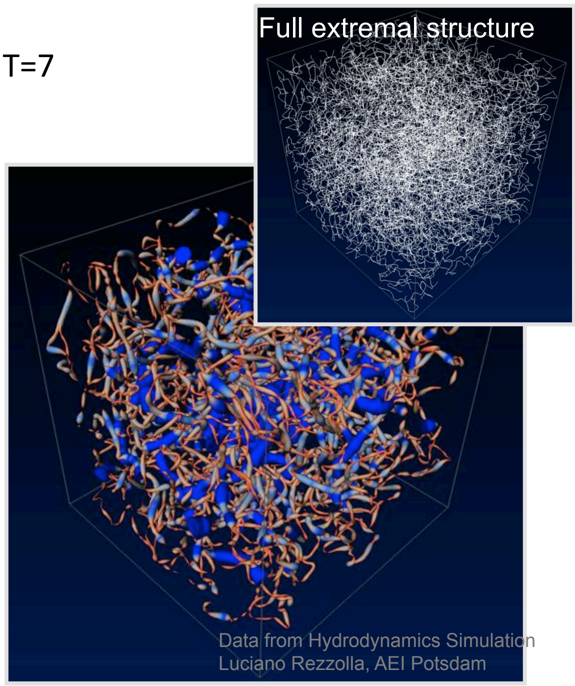
Extremal structures of scalar vortex identifier for vortex core extraction

6 Examples from flow visualization

T=2

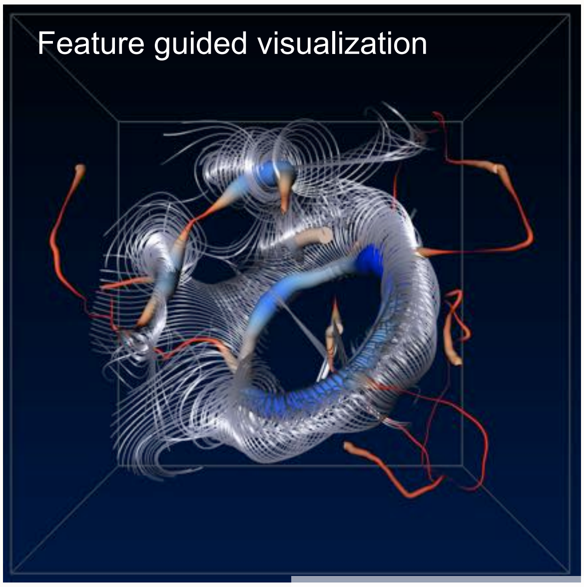


T=7

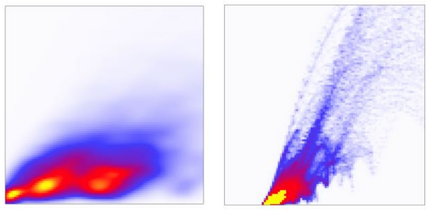


Extremal structures of scalar vortex identifier for vortex core extraction

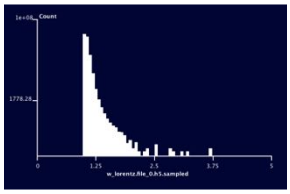
Feature Extraction – Flow Analysis (Vortex Extraction)



Statistical Analysis + Exploration
E.g. Scatterplots,



Histograms

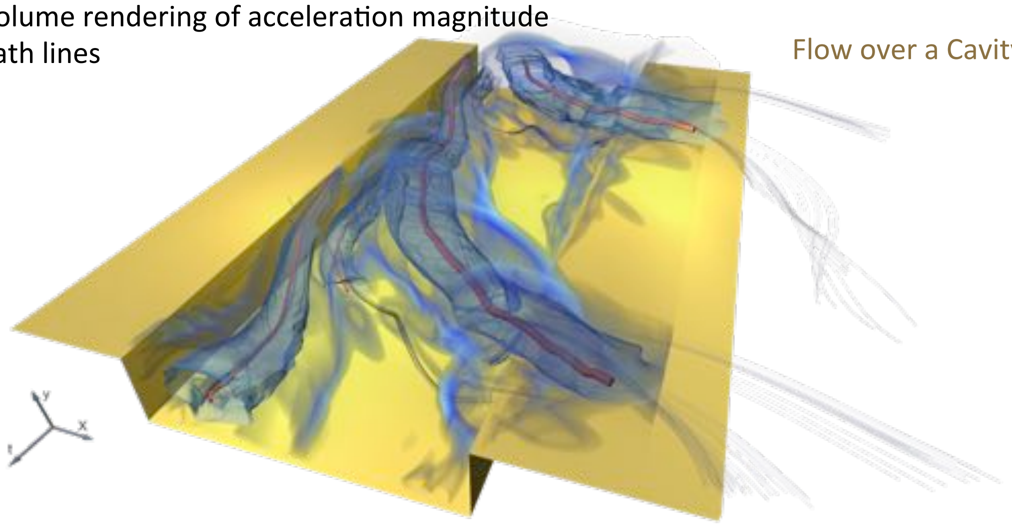


,

Topological tracking of vortices and vortex regions

- Vortex core-lines (red)
- Associated vortex regions (blue)
- Volume rendering of acceleration magnitude
- Path lines

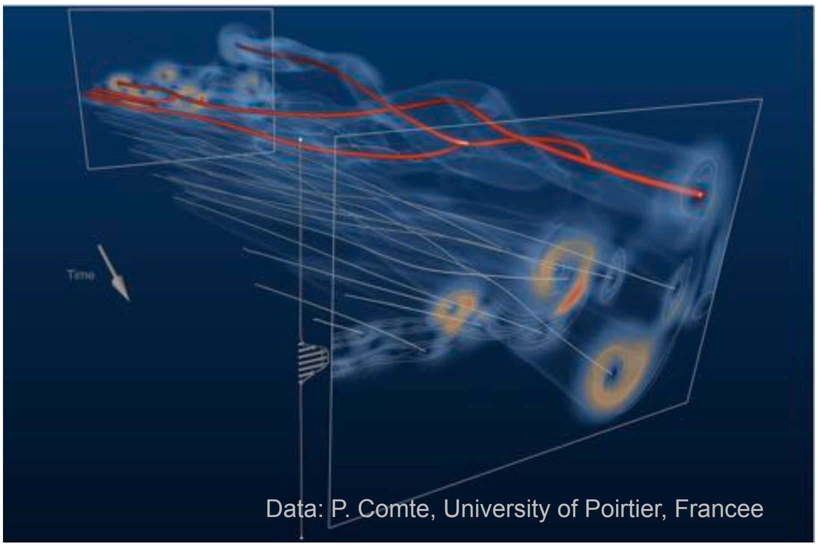
Flow over a Cavity



Two-dimensional Time-dependent Vortex Regions based on the Acceleration Magnitude
Kasten, Reininghaus, Hotz, Hege, TVCG (2011)

6 Examples from flow visualization

Topological tracking of vortices in 2D flow simulation data provides explicit merge trees for the development of vortices



Vortex Merge Graphs in Two-dimensional Unsteady Flow Fields, Kasten, Noack, Hege, Hotz, Proceedings of Eurovis Short Papers, 2012

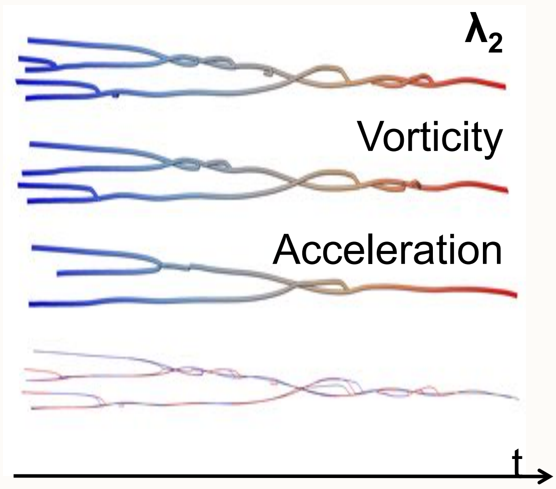
II Topological methods for visualization – Scalar field topology

6 Examples from flow visualization

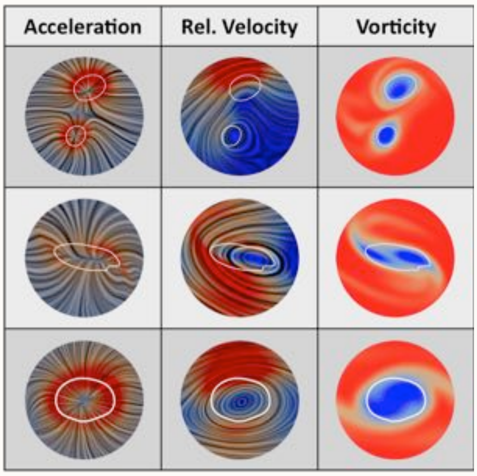
Selected vortex Merge Graph

Comparison of different feature identifiers

merge trees   color – time



Exploration of the neighborhood of vortex core



Analysis of vortex merge graphs Kasten, Zoufahl, Hege, Hotz; Vision, Modeling, and Visualization (VMV'12), 2012

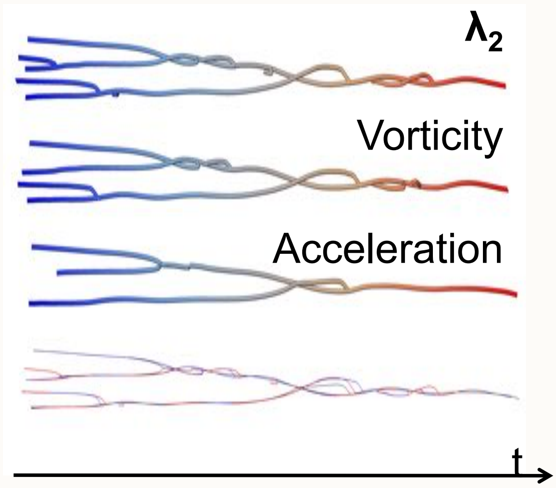
6 Examples from flow visualization

Selected vortex Merge Graph

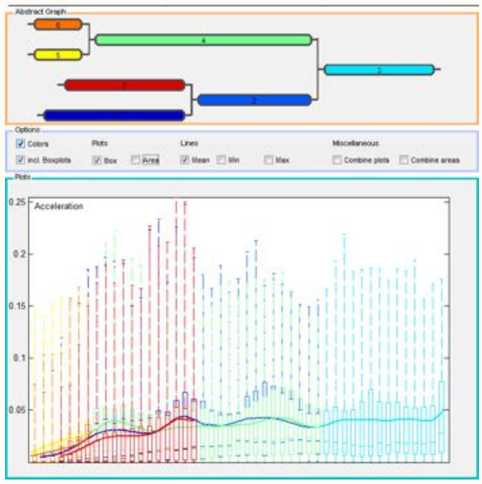
Comparison of different feature identifiers

merge trees

color – time

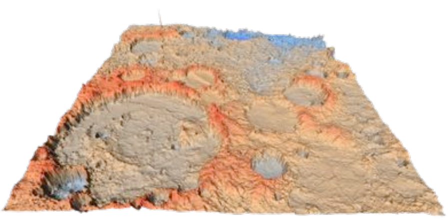


Exploration of the neighborhood of vortex core

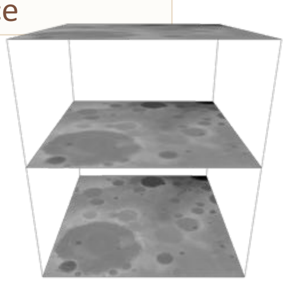


6 Examples other importance measures

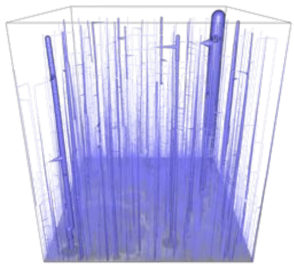
Scale-space Based Persistence



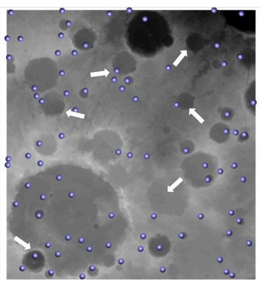
Elevation map of a region on Mars



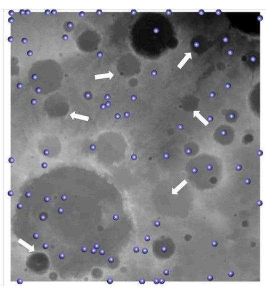
Scale-space



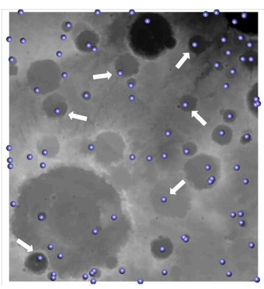
Evolution of minima in scale-space



Homological persistence



Scale-space lifetime



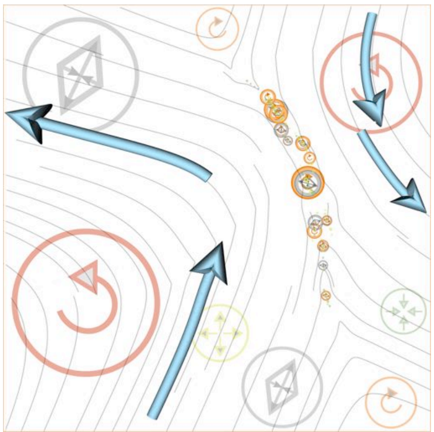
Scale-space persistence

A Scale Space Based Persistence Measure for Critical Points in 2D Scalar Fields Jan Reininghaus, Kotava, Günther, Kasten, Hagen, Hotz, TVCG, 2011

6 Examples – topology for automatic sketch generation

Illustrative representation based on topology

Simulation of co-seismic displacements

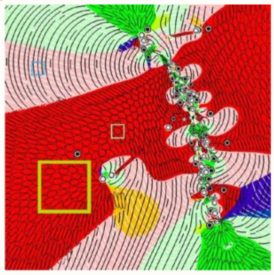


Automatically generated sketch

- Context representation as background
- Strongly expressed features as foreground

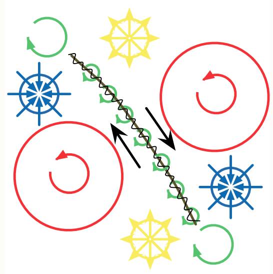
Hybrid visualization (Chen2011)

- Hyper-streamlines
- Elliptical glyph



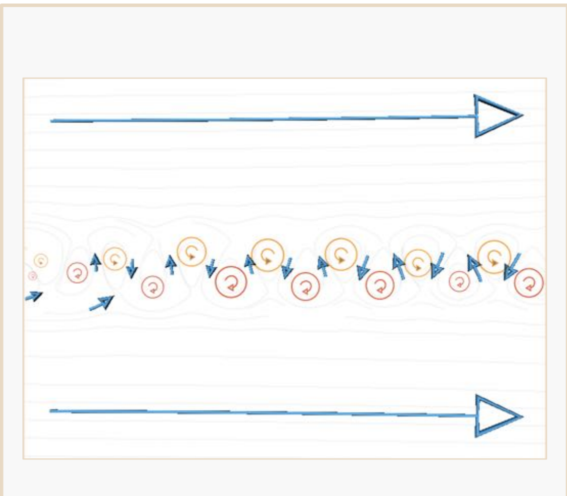
Hand drawn sketch

- drawn by domain experts on basis of the visualization

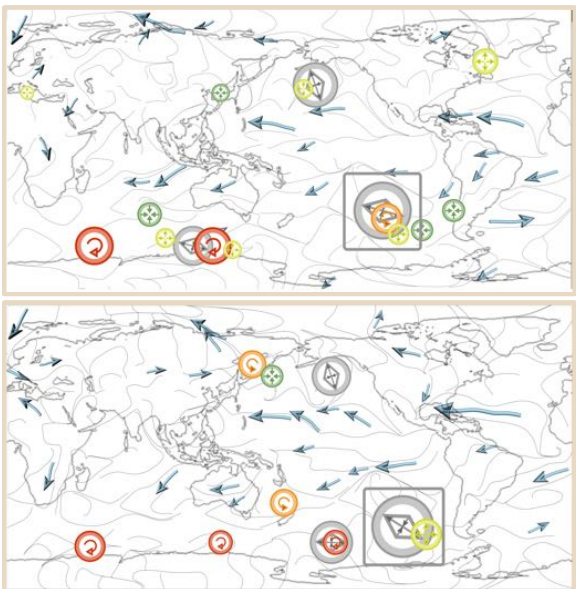


Automatic, Tensor-Guided Illustrative Vector Field Visualization, Cornelia Auer and Jens Kasten, Kratz, Zhang, Hotz, IEEE PacificVis Conference, 2013

6 Examples – topology for automatic sketch generation



- Time dependent simulation of wind in climate model (Two times steps)



Scalar Field Topology in Visualization – Wrap up

Topology – rubber sheet geometry

- Many visualization methods can be built on topological analysis
- **Contour-tree** is a topological representation keeping track of the number of contours, not other topological changes are considered.
- **Extremal structures** generate a skeleton of the data containing much of the relevant information
- **Segmentation**
- …..

Scalar Field Topology in Visualization – some notes

Topology – practical applications

- **Robust and efficient extraction** of topology as well as the use in specific applications is an active research area
- **Importance measures** and simplification are essential for usability
- Sometimes it is necessary to relax the strict mathematical context to reach practical solutions
- We just scratched the surface of the topic

Overview

I. Introduction
II. Scalar field topology
III. Vector field topology
- **1 Some basic vector visualization methods**
- **2 Motivation**
- **3 Introductory Example**
- **4 Basic concepts**
- **5 Linear Vector fields**
- **6 Outlook**
- **7 Application for streamline placement**

IV. Tensor field topology

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1 Some basic vector visualization methods

$$\mathbf{v}: D \to \mathbb{R}^3, \quad \mathbf{x} \mapsto \mathbf{v}(\mathbf{x}), \quad D \subset \mathbb{R}^3$$

1 Some basic vector visualization methods

Integral curves

- Streamlines (Integral line)
  - Everywhere tangential to vector field at fixed time

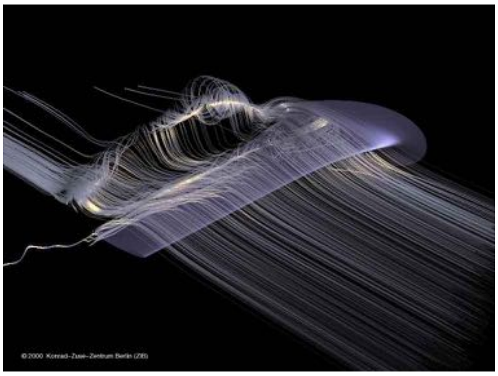


Image: Tino Weinkauf, ZIB, Amira

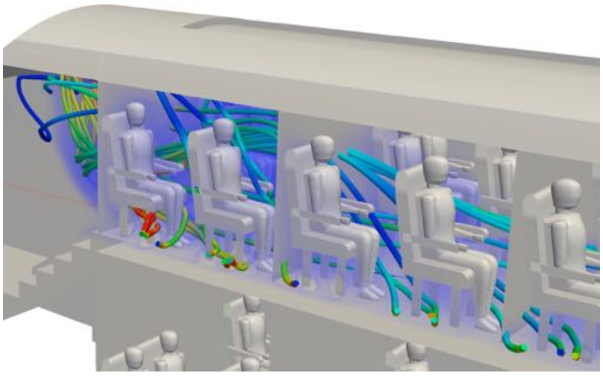


Image: Markus Flatken, DLR, Paraview

1 Some basic vector visualization methods

Integral curves

- Streamlines (Integral line)
  - Everywhere tangential to vector field at fixed time

Without picture

- Pathlines (Integral line)
  - Trajectories of mass-less particles

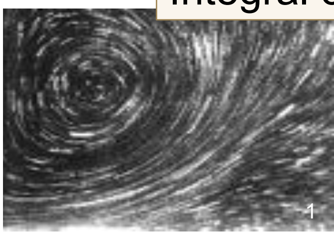
1

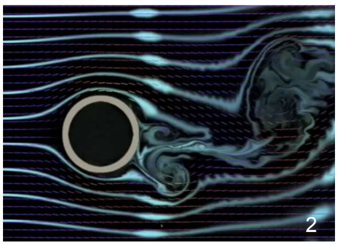
- Streaklines
  - Trace of ink injected at a fixed position

2

- Timelines
  - Propagation of lines or surfaces of mass-less particles

3

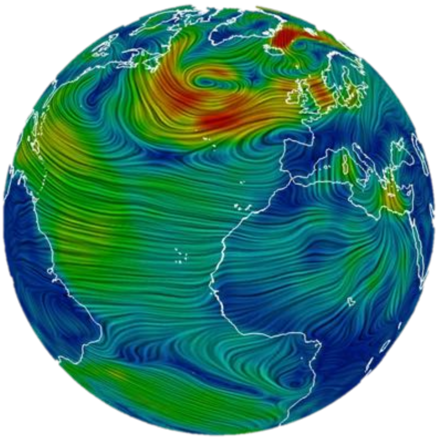


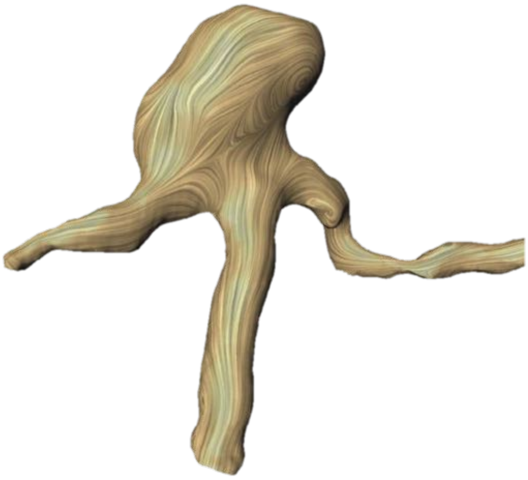


1 Some basic vector visualization methods

Textures

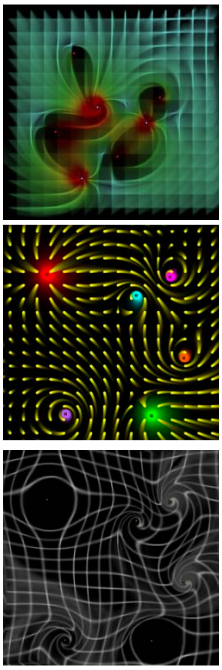
- Line integral convoltion

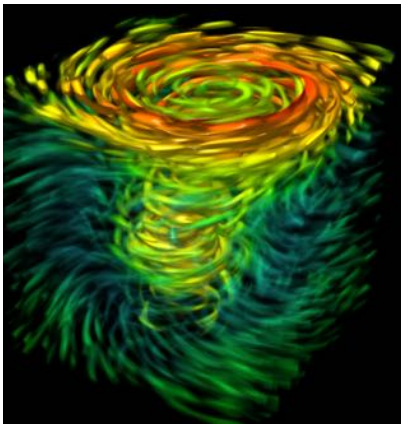


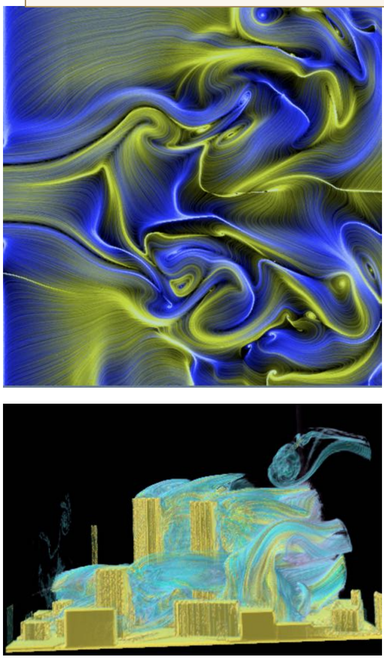


1 Some basic vector visualization methods

Textures - advaction





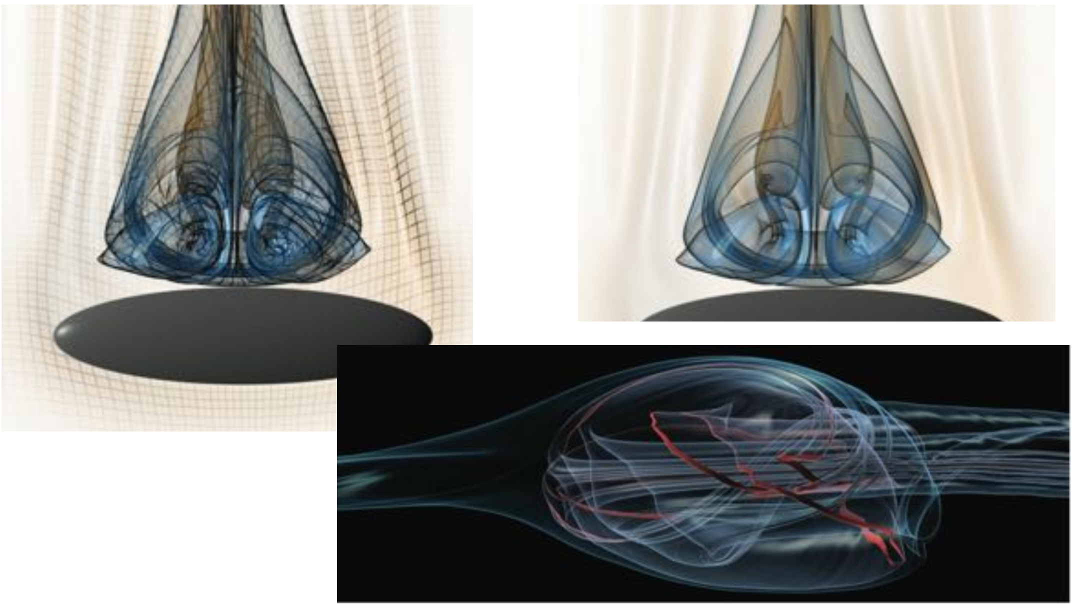


Images: van Wijk, J. J.,Eindhoven Weiskopf, Uni Stuttgart Park, UC Davis

1 Some basic vector visualization methods

Streamsurfaces

Illustrative enhancement, texture mapping



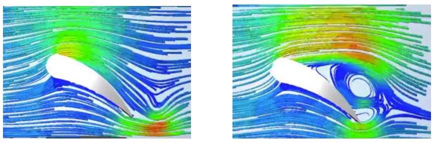
Hummel, M.et al., IRIS: Illustrative Rendering of Integral Surfaces
IEEE Transactions on Visualization and Computer Graphics (Vis'10), **2010**, *16*, 1319-1328

2 Motivation – Why more detailed analysis?

2 Motivation

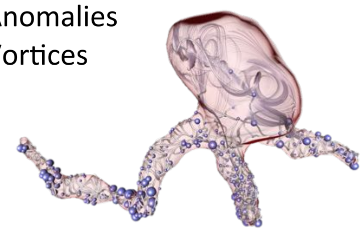
Typical Questions

Flow around a body (e.g. car, airplane)
- Vortex formation
- Flow separation



Wing Flow Separation (Stall)

Medicine – flow in blood vessels
- Anomalies
- Vortices



Vortex in blood flow in aneurysm
Scalar topology. Kasten (ZIB)

Combustion and fuel injection into engines

Pollution distribution of particles in the atmosphere or water systems

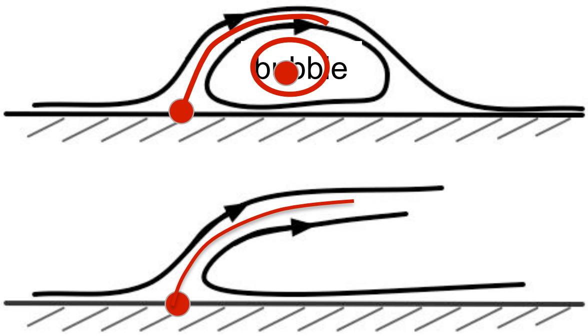
→ Mixing process

Mixing of a fluid – color pH value of fluid.
CAP Arts of Physics, vis thymol blue.

2 Motivation

Hand drawn sketches

Anticipated typical flow structures
- Relation of vortex formation and separation?
- Characteristic singularities of the flow field?

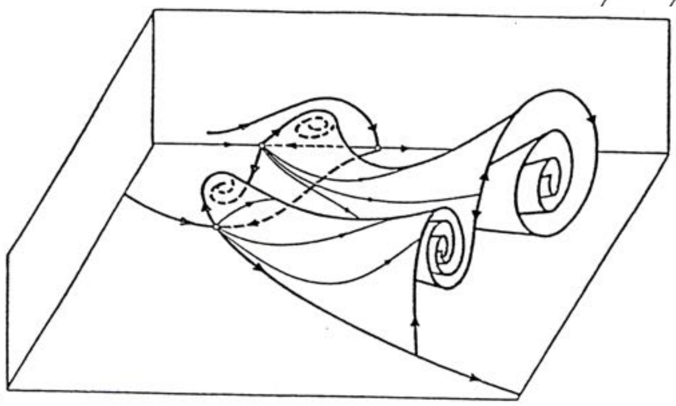


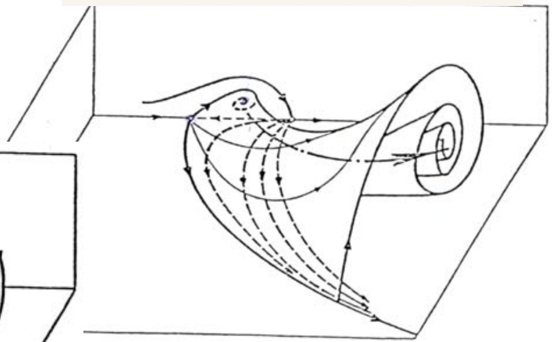
Often recirculation zones form behind obstacles
Does separation cause recirculation?

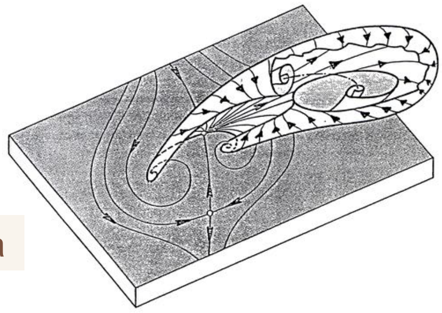
2 Motivation

Hand drawn sketches

Separation and vortex formation





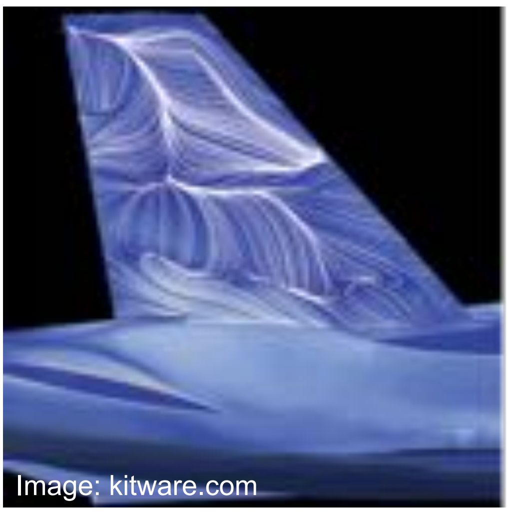


Goal: Generate similar images from data

Images: Dallmann, German Aero Space, DLR

2 Motivation

Obviously there is some structure in most vector field data.
Feature extractions tries to make this structure explicit.

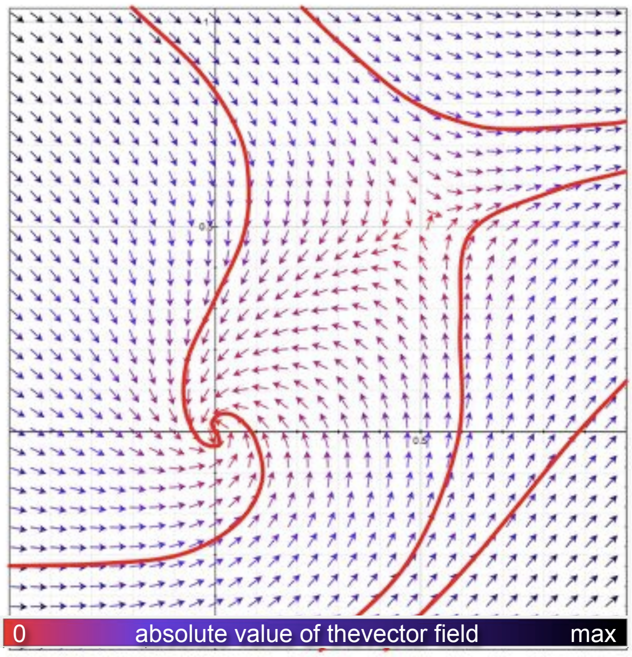


3 Basic concept

3 Basic concept

A few streamlines

What about the other streamlines?
Can we tell where they go?

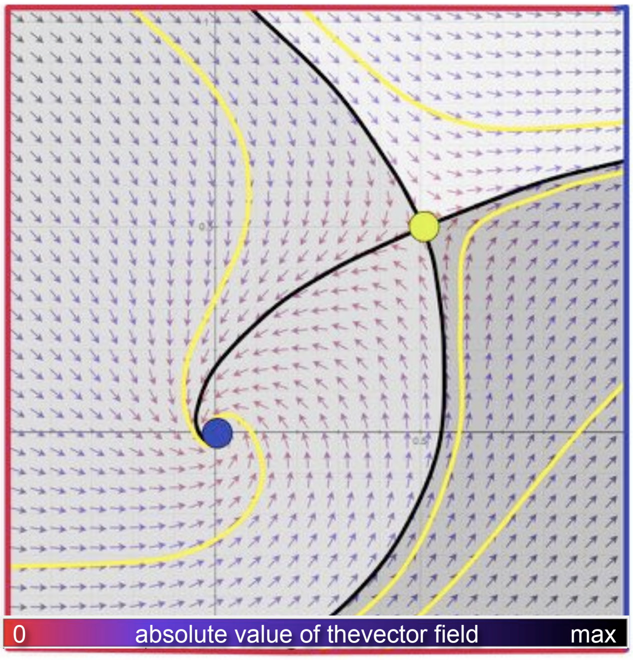


3 Basic concept

A few streamlines

What about other streamlines, can we tell where they go?

→ Topology answers this question for **ALL** streamlines.

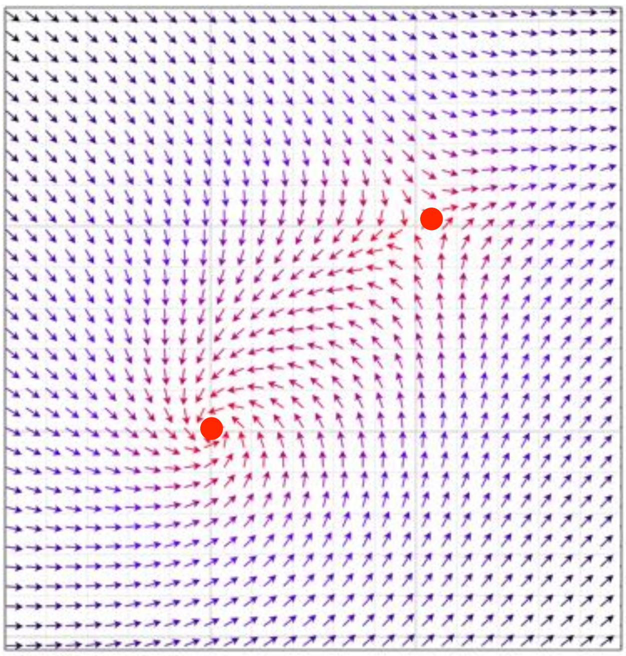


3 Basic concept

Vector field topology

Ingredients

1. Critical points – zeros
   - Positions $\mathbf{v}(x,y) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

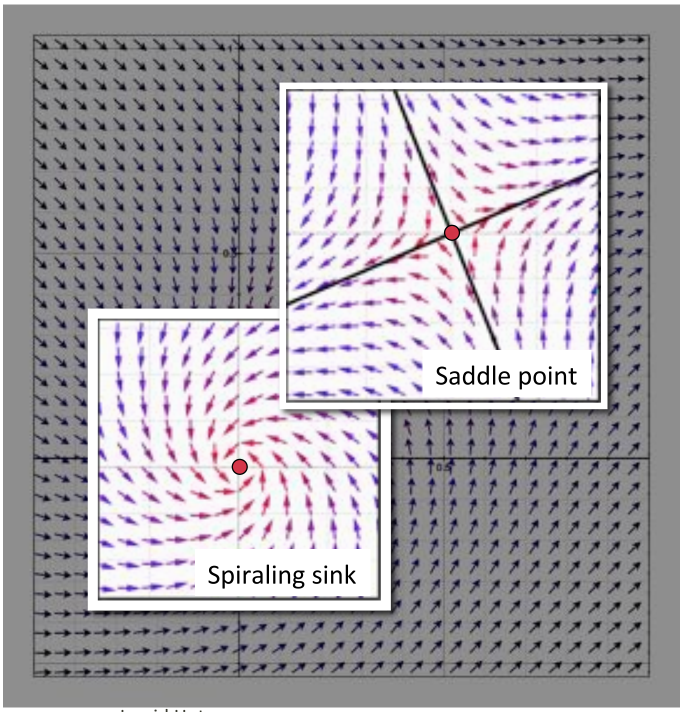


3 Basic concept

Vector field topology

Ingredients

1. Critical points – zeros
   - Positions
   - Classification



III Topological methods for visualization – Vector field topology

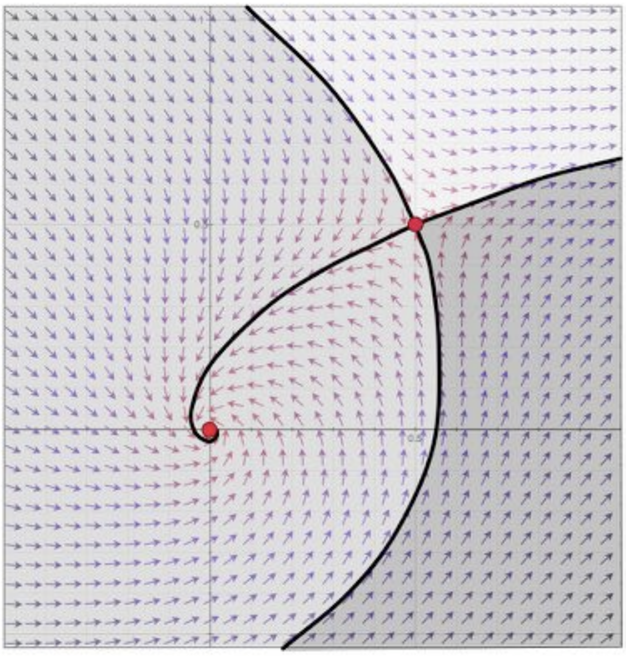
3 Basic concept

Vector field topology

Ingredients

1. Critical points – zeros
   - Positions
   - Classification
2. Separatices

→ Segmentation of domain into areas of similar streamline behavior



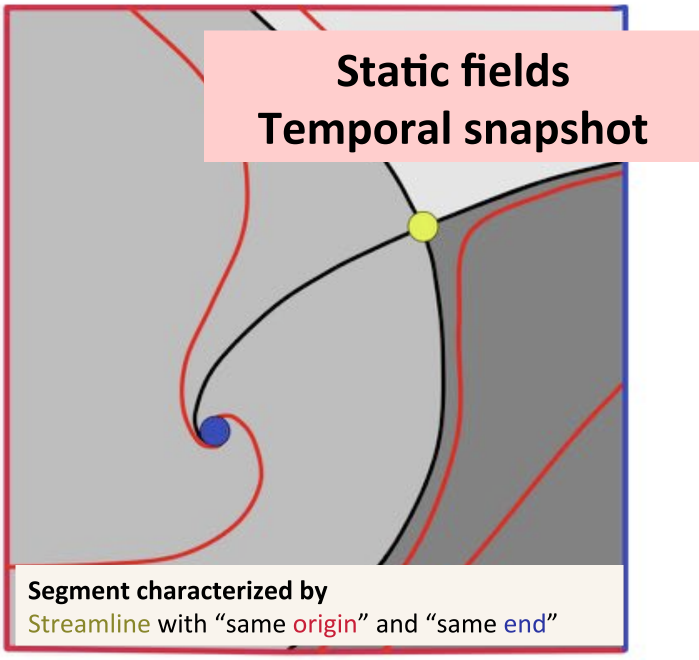
3 Basic concept

Vector field topology

→ Segmentation of domain into areas of similar streamline behavior

Based on ideas from Poincaré over qualitative investigations of differential equations (19th century) ,

Theory of dynamical systems



Reference: Helman, J. & Hesselink, L., Representation and Display of Vector Field Topology in Fluid Flow Data Sets *Computer,* **1989**, *22*, 27-36

III Topological methods for visualization – Vector field topology

3 Basic concept

Streamline origin / destination

→ Define **start-set / end-set** for every streamline

Idea: Every point *P* is assigned to the start/end set of its streamline

Definition

α-limit (ω-limit) set to streamline c_p through point P
for vector field $\mathbf{v}: D \rightarrow \mathbb{R}^n$

$$\mathrm{A}(c_p) := \left\{ q \in D \mid \exists (t_n)_{n=0}^{\infty} \subset R \text{ with } \lim_{n\to\infty} t_n = -\infty, \text{such that } \lim_{n\to\infty} c_p(t_n) = q \right\}$$

$$\Omega(c_p) := \left\{ q \in D \mid \exists (t_n)_{n=0}^{\infty} \subset R \text{ with } \lim_{n\to\infty} t_n = \infty, \text{ such that } \lim_{n\to\infty} c_p(t_n) = q \right\}$$

3 Basic concept

→ **The topological graph or skeleton** of a planar 2D vector field consists of all limit sets and separatrices

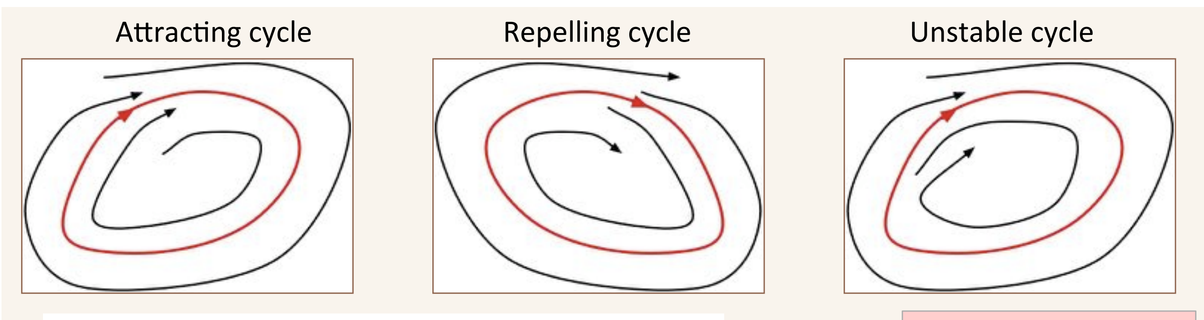
3 Basic concept

Limit Sets

Critical points: Zeros of the vector field (Local definition)

Alternative terms: singularities, singular points, zeros, stagnation points

Closed orbits: attracting or repelling (No local definition)



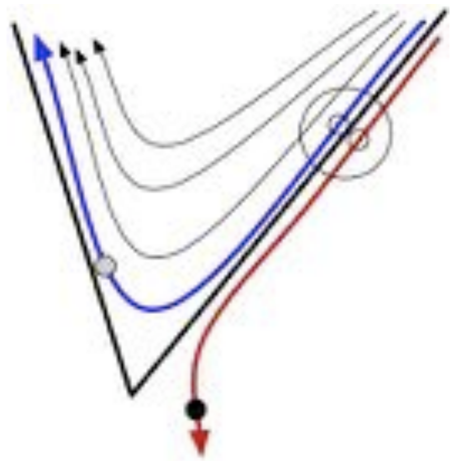
Extracting closed streamlines robustly is a challenging task

There are also boundary contributions

3 Basic concept

Separatrices

- Limiting curves – Separatrices connect the critical points



3 Basic concept

Linear Vector Fields

Why linear vector fields?

- Linear vector fields can be analyzed relatively easily
- More complex vector fields can be first order approximated by linear vector fields (use Jacobi-Matrix).
- On tetrahedral grids with linear interpolation we have linear fields

3 Basic concept

Linear Vector Fields

A **linear vector field** is given by

$$\mathbf{v}: D \rightarrow \mathbb{R}^n$$
$$\mathbf{v}(\mathbf{x}) = \mathbf{A} \cdot \mathbf{x} + \mathbf{b}$$

- A matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$
- A vector $\mathbf{b} \in \mathbb{R}^n$

The matrix **A** can be used to classify the behavior of the vector field in the neighborhood a critical point.

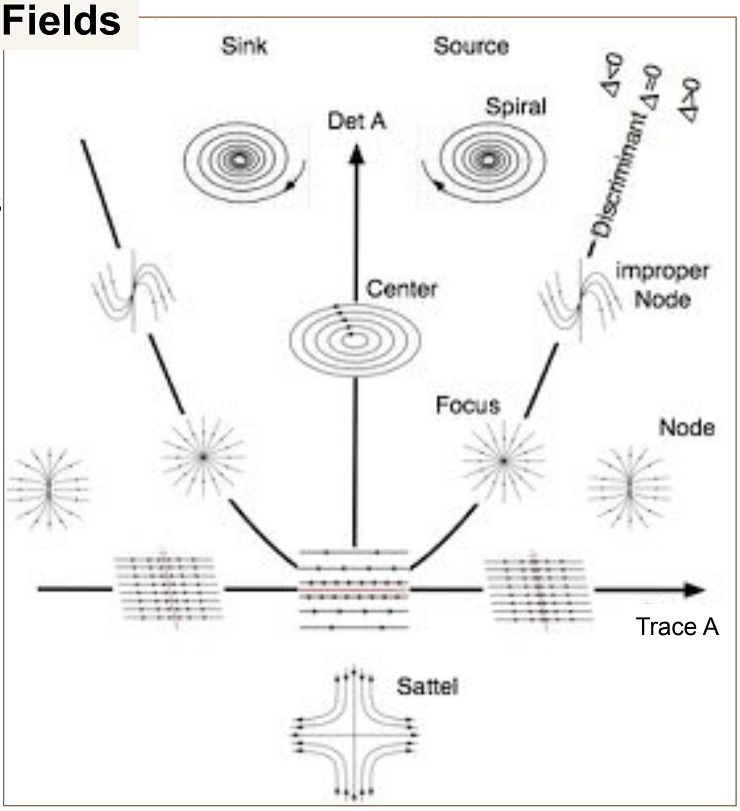
[Nielson, Tools for Computing Tangent Curves and Topological Graphs for Visualizing Piecewise Linearly Varying Vector Fields, in Scientific Visualization Overviews, Methodologies, Techniques,

Linear Vector Fields

Classification of linear critical points

$$\lambda_{1/2} = \frac{\mathrm{tr}(\mathbf{A})}{2} \pm \sqrt{\Delta}$$

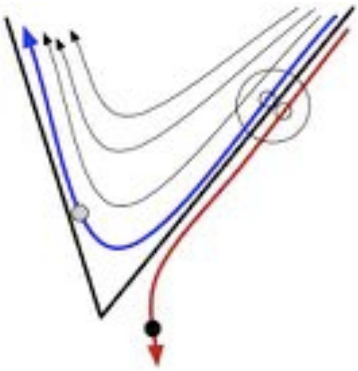
$$\underbrace{\frac{1}{4}\mathrm{tr}^2(\mathbf{A}) - \det \mathbf{A}}_{\text{Discriminant } \Delta}$$

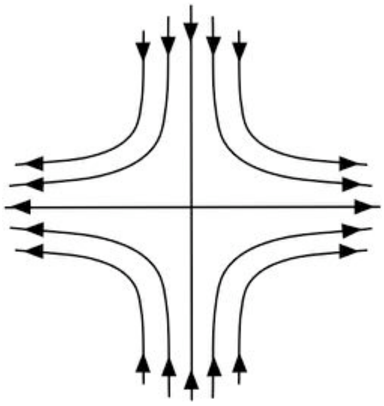


3 Basic concept

Separatrices of linear fields

- Separatrices are streamlines entering/leaving the saddles in direction of the eigenvecotrs of the matrix **A**





III Topological methods for visualization – Vector field topology

3 Basic concept

General Vector Fields

Linear saddle point

Non-linear saddle point

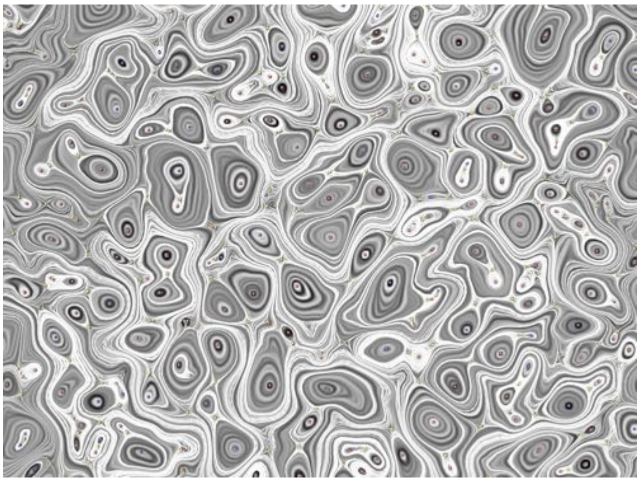
4 Outlook - remarks

- Simplification
- 3D Fields
- Discrete vector field topology

4 Outlook - remarks

Numerical Computation Challenges

- No simple way to deal with noisy data
- High feature density
- Many computational parameters



4 Outlook

Simplification and scaling of the topologic structure

→ Strategies to consistently merge critical points

→ So far no consistent theory, mostly heuristics, many numerical challenges

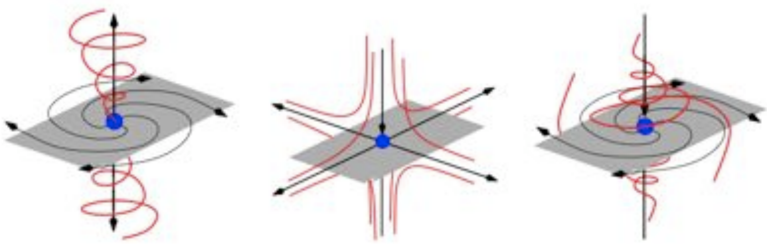
Image: PhD Thesis, Xavier Tricoche, Purdue University

4 Outlook

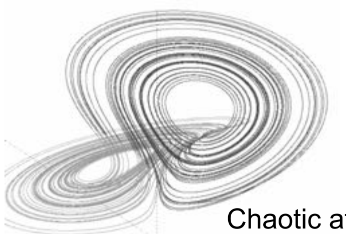
3D Topology – more structures possible than for 2D

→ Separating surfaces and characteristic lines

→ New possible limit sets: chaotic attractors, surfaces



(a) Repelling spiral (b) Saddle point (c) Spiral saddle



Chaotic attractor

4 Outlook

3D Topology

Example: electrostatic field of a Benzol molecule

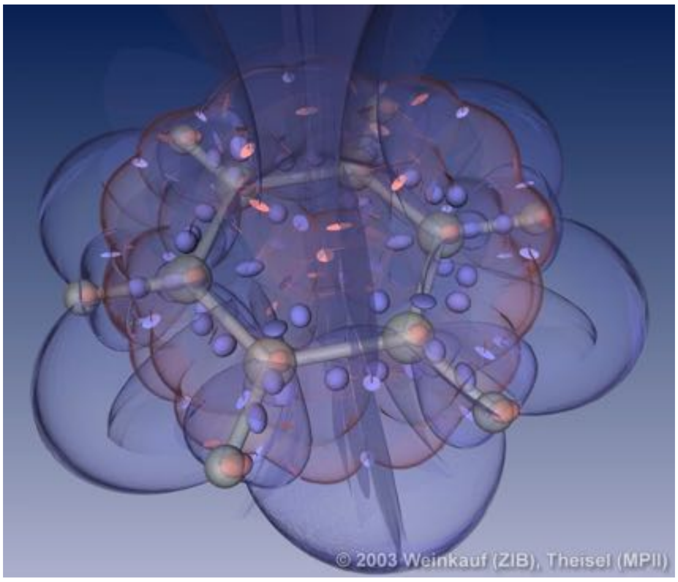
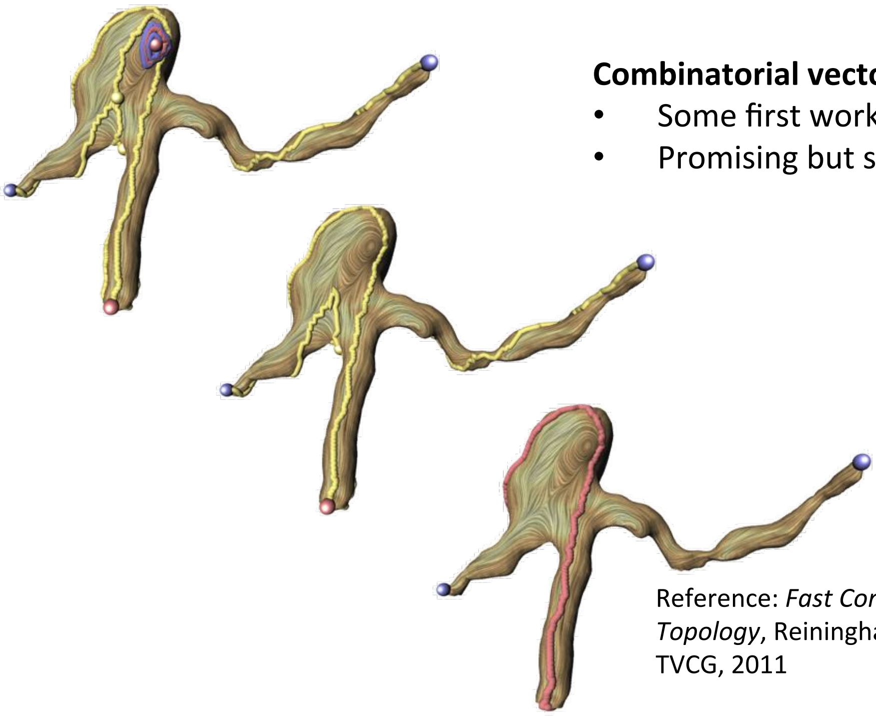


Image: Tino Weinkauf, ZIB, Amira

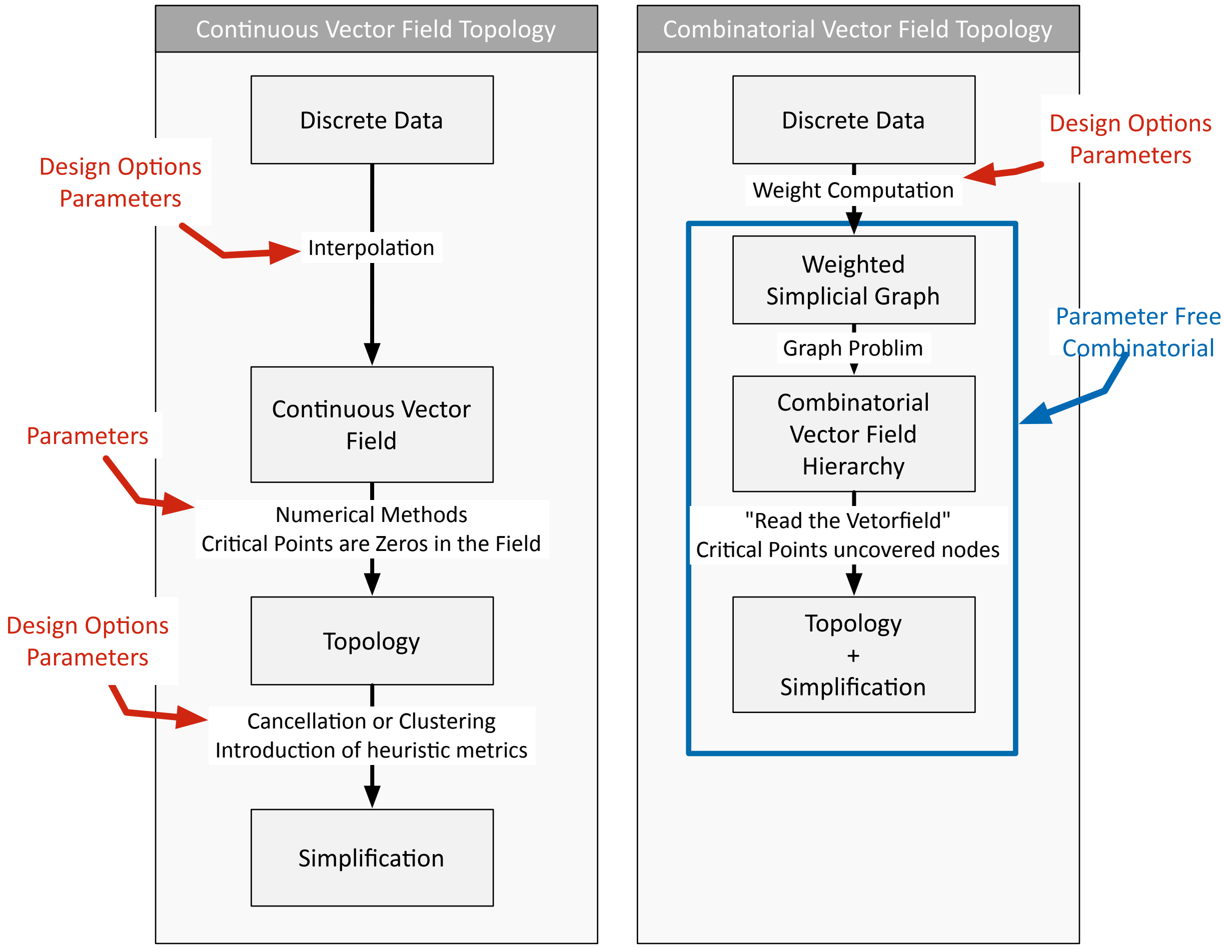
4 Outlook



Combinatorial vector field topology

- Some first work
- Promising but still a long way to go

Reference: *Fast Combinatorial Vector Field Topology*, Reininghaus, Löwen, Hotz TVCG, 2011



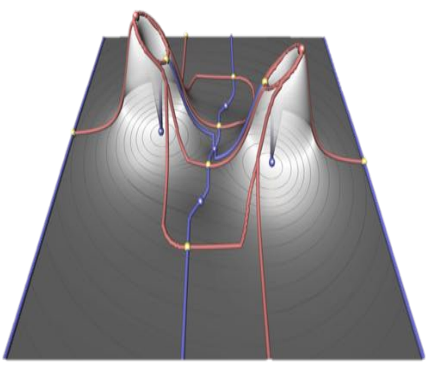
4 Outlook

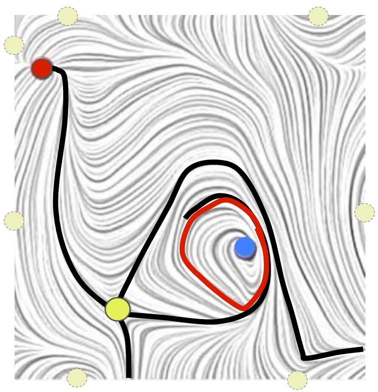
	Continuous Vector field topology		Combinatorial Vector field topology	
Geometric Embedding	+	Smooth streamlines High spatial precision	-	Follows edges of the graph
Topological Consistency	-	Cannot be guaranteed	+	Always guaranteed (Morse Inequalities)
Robustness	-	Problems with noise and high feature density	+	Importance measure with theoretical guaranties
Simplicity	-	Many parameters	+	Almost parameter free
Runtime	+	Reasonable	o	Is getting better

5 Vector vs. scalar field topology

5 Vector vs. scalar field topology

	Scalar fields	Vector fields
Origin	Morse theory	Dynamical systems
Critical points	Maxima, Minima, Saddles	Sources, Sinks, Saddles
Closed Orbits / Cycles	no	yes
	Special case of vector fields: gradient vector field, **rotation free**	





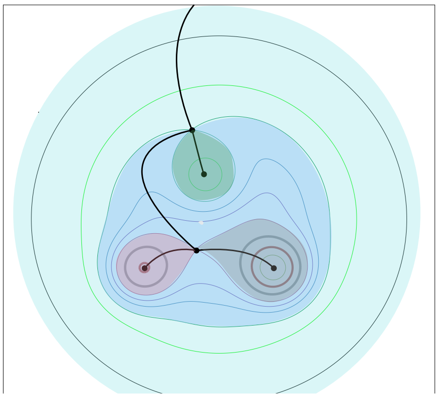
5 Vector vs. scalar field topology

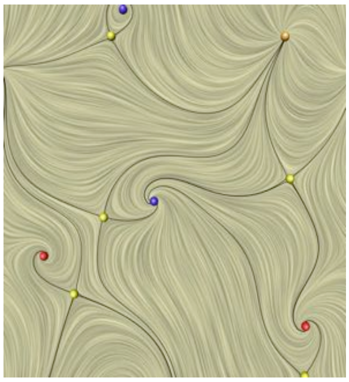
Scalar fields

Contour Tree
- Equivalence classes for contours (orthogonal to gradient lines/ streamlines)

Vector fields

Topological graph
- Equivalence classes for streamlines





Black lines intersect all contours of the data set, they have no topological significance

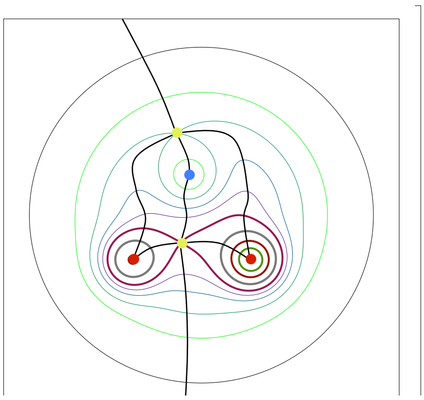
5 Vector vs. scalar field topology

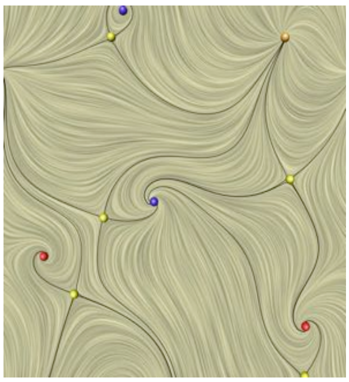
Scalar fields

Contour Tree
- Equivalence classes for contours (orthogonal to gradient lines/ streamlines)

Vector fields

Topological graph
- Equivalence classes for streamlines





Topology of gradient vector field with separtrices and critical points.

6 Application: Streamline Placement

6 Application: Streamline Placement

Typical placements

- Interactive choice of single start points
- Start streamlines in all mesh vertices
- Start streamlines at random positions

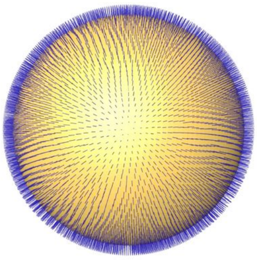
→ Often very inhomogeneous coverage

Goals

- Coverage
- Uniformity
- Continuity
- Highlight features (CPs)

6 Application: Streamline Placement

Dual Seeding designed for tangent vector fields (Roswanow, et. al)



Surface with normals

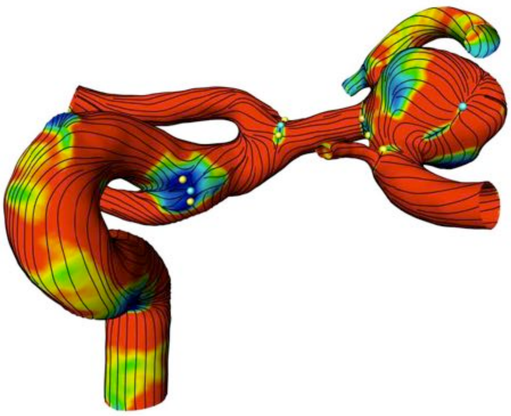
Tangent vector field

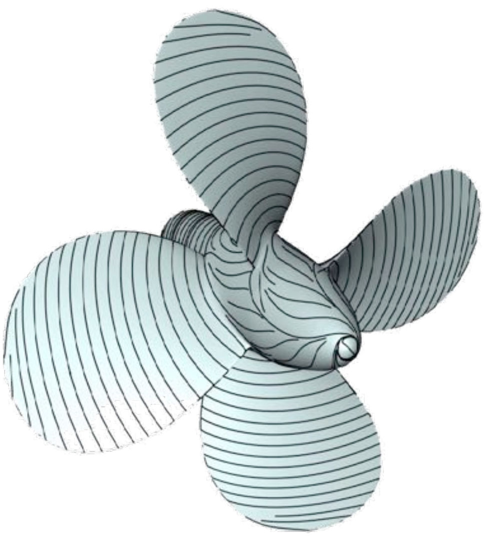
Streamline placement

Flexible streamline density

Images: Roswanow, ZIB

6 Application: Streamline Placement





Images: Olufemi Rosanwo (ZIB,Amira)

Some Remarks

However vector field visualization never really took off

Possible reasons

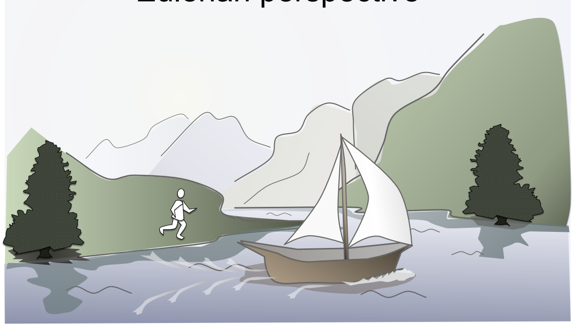
- No robust extraction methods
- No consistent simplification strategy
- Structures for 3D can become very complicated
- Interpretation requires having the interest and time to become involved, and this are mostly scientist doing basic research

Some Remarks

Maybe the most severe limitation is
Vector field topology is **not directly applicable to unsteady vector fields**

- What is the meaning of limit sets?
- Only finite time span for flow available
- Not invariant with respect to change in reference frame

Eulerian perspective

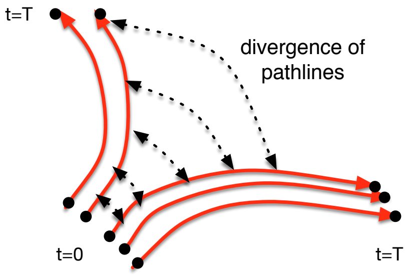


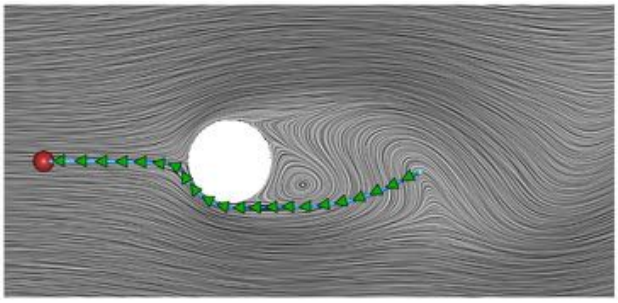
Lagrangian perspective

Alternatives for unsteady flow fields – Lagrangian view

Time dependent features – highlight separating structures

- Lagrangian coherent structures, Finite time Lyapunov Exponent (FTLE)
- **Somehow** generalization of **some concepts** of vector topology to time-dependent fields (**not strictly**)





References:

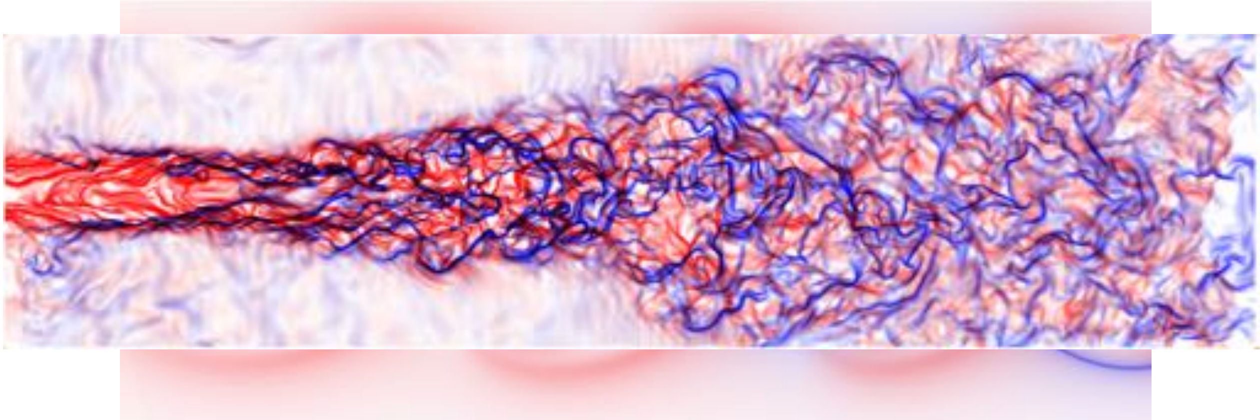
- *Distinguished material surfaces and coherent structures in three-dimensional fluid flows,* George Haller, Phys. D, 2001
- *Localized Finite-time Lyapunov Exponent for Unsteady Flow Analysis* (inproceedings), Kasten, Petz , Hotz, Noack, Hege, Vision, Modeling, and Visualization (VMV'09), 2009

III Topological methods for visualization – Vector field topology

Alternatives for unsteady flow fields – Lagrangian view

Feature-extraction

Emphasize divergent and convergent flow behavior



Method: 'Finite Time Lyapunov Exponent'

Data: Van Kármán vortex street, Mutschke TU Dresden, Images: Jens Kasten, ZIB, Amira

Summary

- Topology provides many powerful concepts for feature based vis
- Scalar field topology is a rapidly developing field, unfortunately not yet available in commercial tools