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Modelling of Prices Using the Volume in the Norwegian Regulating Power Market

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***Abstract*—A statistical model of the regulating market based on the regulating volume is proposed. The modelling process is divided into two steps; a long term and a short term study. The long term study is based on recorded data for 5 years. This analysis provides a statistical model of regulating prices and volumes for the whole market for the considered period. The combination of the long term model with expected regulating states and volumes is used in order to generate short term scenarios of the regulating market. The regulating state determination uses a Seasonal Auto Regressive Integrated Moving Average (SARIMA) process. The regulating volume scenarios are generated by using the statistical properties of the regulating volume based on recorded data. The proposed model is based on data from southern Norway and the result is a model estimating the regulating prices using the estimated regulating volumes. The resulting model makes it possible to estimate regulating market prices under changing conditions, like those occurring when different national markets are integrated.**

***Index Terms* – Norwegian regulating power market, Regulating market integration, Regulating prices and volumes, Time series analysis.**

I. Nomenclature

pr_{reg}	Regulating price
vol_{reg}	Regulating volume
pr_{spot}	Spot market price
Δpr	Difference between regulating price and spot price
η_{up}, κ_{up}	Linear regression coefficients for upward regulating
$\eta_{down}, \kappa_{down}$	Linear regression coefficients for downward regulating
μ_{up}, μ_{down}	Location parameter of the error's EV distribution for up- and downward regulation
$\sigma_{up}, \sigma_{down}$	Scale parameter of the error's EV distribution for up- and downward regulation
μ_{no}	Mean value of the error's normal distribution for no regulation
σ_{no}^2	Standard deviation of the error's normal distribution for no regulation
$\epsilon_{up}, \epsilon_{no}, \epsilon_{down}$	Error terms
p, d, q	Hourly parameters
P, D, Q	Seasonal (daily) parameters
B	The backward shift operator i.e. $Bx_k = x_{k-1}$
S	Season length i.e. 24 hours.
Φ_i	Coefficients of seasonal Auto Regressive (AR) polynomial $i = 1 \cdots P$
ϕ_i	Coefficients of AR polynomial $i = 1 \cdots p$
Θ_i	Coefficients of seasonal Moving Average (MA) polynomial $i = 1 \cdots Q$
θ_i	Coefficients of MA polynomial $i = 1 \cdots q$
∇^d	Hourly difference operator of order d
∇_S^D	Seasonal difference operator of order D
Z_t	White noise
σ_{WN}	Standard deviation of Z_t
$\mu_{up}^{vol}, \mu_{down}^{vol}$	Location parameter of the volume's GEV distribution
$\sigma_{up}^{vol}, \sigma_{down}^{vol}$	Scale parameter of the volume's GEV distribution
$k_{up}^{vol}, k_{down}^{vol}$	Shape parameter of the volume's GEV distribution

II. Introduction

THE European Union Electricity Market Directive 96/92/EC and the 2003 Directive 54/EC set goals for the gradual opening and integration of the electricity markets in the member states. Regulation 1228/2003 for the first time also explicitly addresses cross-border issues [1]. At the same time, the need for more sustainable power production has resulted in a rapidly increasing share of wind power in Northern Europe, notably in Denmark and Germany, but also in the Netherlands. High shares of wind power induce a need for more flexible regulation resources, and Norwegian hydro power can be a highly useful contribution, taking into account the increasing connection capacities between Norway and continental Europe.

An integration of regulating markets can facilitate the mutual procurement of regulating resources and is a way of using existing cross border capacity more efficiently. From an economic point of view, the framework for an efficient regulating market integration should maximize social welfare. The integration of regulating markets will have a significant effect on the regulating volumes in the individual markets and will therefore influence the regulating price. In order to study the effects on prices and social welfare, it is necessary to model the effect of the regulating volumes on the regulating market. So to assess the potential effects of integrating Nordic and continental European regulating markets it is important to study the characteristics of the present individual markets before integration.

The current Nordic electricity system characteristics and the newly commissioned NorNed HVDC cable suggest that southern Norway has the highest potential to be an exporter

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of regulating resources to the European Continent. Thus the first study of the individual regulating markets is done on the regulating market behaviour in southern Norway.

There are many studies of NordPool's spot market prices, but only a few on the Nordic regulating market. Fleten and Pettersen [2] used a method called generation of moment matching scenarios. Olsson and Söder [3] proposed a model based on seasonal auto regressive integration moving average (SARIMA) and Markov processes. Both papers forecast the regulating price without taking into account the regulating volume. Skytte [4] did an econometric analysis of the regulating market, creating a linear model that takes into account the influence of the regulating volume. In this paper a linear model is proposed which is extended by error terms. Further on a SARIMA process is introduced in order to generate regulating price scenarios. The model can be used to estimate regulating market prices in the case of an increasing demand for regulation, e.g. caused by demand over the HVDC interconnection or by increased utilization of wind power.

III. Model Formulation

The proposed model is developed in two steps. First a long term statistical modelling of the market is designed, which is described in part III-A. The long term statistical model describes the normal behaviour of the regulating market, especially the regulating price/regulating volume dependence. Normal behaviour here means that extreme events that can occur are neglected. Extreme events are unusual high regulating prices as well as unusual high regulating volumes. Thus only those hours of the recorded market data are considered where the price and volume are within three times the standard deviation.

In order to capture the inherent time dependence of the regulating market a second step, the short term modelling is done, which is described in part III-B. The short term model results in the forecast of the system's regulating state and subsequently regulating volume scenarios, which are used as the input for the long term model.

The model is developed based on recorded market data of southern Norway (NO1). As there are interconnections to neighbouring areas, like northern Norway, Sweden and through HVDC lines to Denmark, NO1 is not an isolated area. On the contrary the whole Nordic area is a single regulating market thus regulating prices in southern Norway are influenced by the other areas.

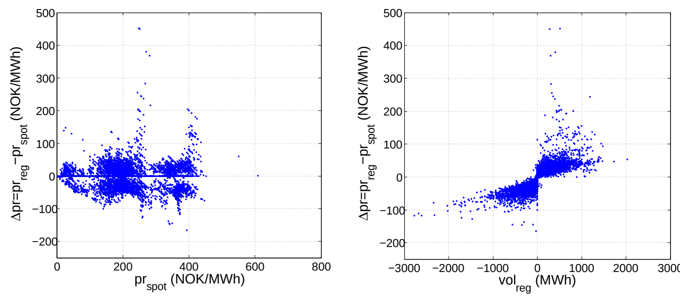
A. Statistical model

In order to develop a statistical model which is based on the regulating volumes an approach similar to the one of Skytte [4] is used. He formulates a model describing the dependence of the regulating price pr_{reg} on the regulating volume vol_{reg} and the spot market price pr_{spot}.

In this paper not the regulating price but the price difference Δpr of regulating and spot market price, as stated in (1), is utilized.

$$\Delta pr = pr_{reg} - pr_{spot} \qquad (1)$$

In Fig. 1a and 1b Δpr is plotted against pr_{spot} and vol_{reg} respectively. These diagrams indicate the correlation of the according values. Δpr and pr_{spot} are not correlated ($\varrho\left(pr_{spot}, \Delta pr\right) = -0.0164$). However there is a significant correlation of Δpr and vol_{reg} ($\varrho\left(vol_{reg}, \Delta pr\right) = 0.7811$). Thus in the further development of the model the influence of pr_{spot} on Δpr is neglected and only the influence of vol_{reg} on Δpr is considered.



(a) spot price (b) regulating volume

Fig. 1. Correlation with difference of spot and regulating price Δpr

The developed model is stated in (2). The model consists of a deterministic and a stochastic part. Further on it is split up into three different states, upward, no and downward regulation.

The deterministic part describes the linear dependence of Δpr on vol_{reg}. Therefore the parameters η_{up}, η_{down}, κ_{up} and κ_{down} are defined, separately for up and downward regulation. The parameters for upward and downward regulation of the deterministic part are estimated by a linear regression using market data records. The linear regression is based on the least mean square method.

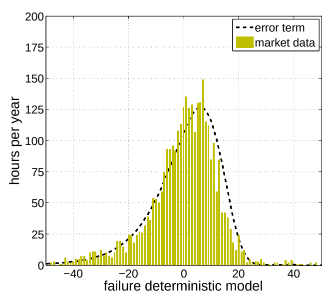
If the results of the deterministic model are compared with the market data records, there will still be a considerable deviation. To model this deviation, the deterministic part is extended by a stochastic part, as stated in (2). In order to extend the model the error terms ϵ_{up}, ϵ_{no} and ϵ_{down} are introduced. For each different regulating state a separate error term is defined. The distributions of the difference between the deterministic model and the market data is shown in Fig.2.

$$\Delta pr_{model} = \left\{ \begin{array}{lll} \eta_{up} + \kappa_{up} \cdot vol_{reg} & + \quad \epsilon_{up} & \text{, if upward regulation} \\ 0 & + \quad \epsilon_{no} & \text{, if no regulation} \\ \underbrace{\eta_{down} + \kappa_{down} \cdot vol_{reg}}_{\text{deterministic}} & + \quad \underbrace{\epsilon_{down}}_{\text{stochastic}} & \text{, if downward regulation} \end{array} \right. \qquad (2)$$

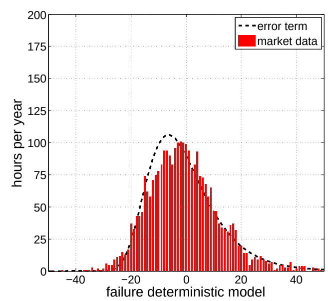
The error terms are described by distribution functions. The definition of the error terms is given by (3) to (5). A detailed description of the distribution functions $EV(\mu,\sigma)$, $C(p)$ and $N(\mu,\sigma^2)$ can be found in (11) to (15) in the appendix.

$$\begin{aligned} \epsilon_{up} &= EV(\mu_{up},\sigma_{up}) \quad (3)\\ \epsilon_{no} &= C(p_{no})\cdot N(\mu_{no},\sigma_{no}^2) \quad (4)\\ \epsilon_{down} &= -EV(\mu_{down},\sigma_{down}) \quad (5) \end{aligned}$$

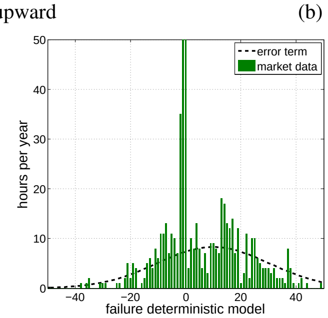
In the no regulating state Δpr is assumed to be zero by the deterministic model. The distribution of the differences in this state can be approximated by a normal distribution with a superimposed peak at zero, shown in Fig.2c. Thus a parameter p_{no} is introduced, defining the probability of $\Delta pr \neq 0$ in the no regulating state. Additionally the parameters μ_{no} and σ_{no}^2 are used to define the mean value and the standard deviation for Δpr in the no regulating state in case of $\Delta pr \neq 0$. A situation with no regulation but the regulating price being different from the spot price can be explained by the impact of regulation in other areas of the Nordic regulating market.



(a) upward



(b) downward



(c) no

Fig. 2. Difference deterministic part and market data and fitted distribution functions of error terms

In the upward and downward regulating state, the distribution of the difference cannot be described by a normal distribution, as it is asymmetric and highly skewed, as depicted in Fig.2a and Fig.2b. Instead, the Extreme Value distribution $EV(\mu,\sigma)$ is used. This distribution function has two free parameters, the location and the scale parameter. μ_{up} and σ_{up} as well as μ_{down} and σ_{down} describe the error terms for upward and downward regulation separately. The parameters are determined by fitting an extreme value distribution function to the distribution of the difference between the deterministic part of the model and the recorded market data. The fitting is done by the maximum likelihood method.

In the statistical model the regulating state is determined by the regulating volume. I.e. upward regulation in the case of $vol_{reg} > 0$, no regulation for $vol_{reg} = 0$ and downward regulation in the case of $vol_{reg} < 0$. The resulting statistical model for southern Norway in the year 2007 is shown in Fig.3. The recorded market data is plotted as dots, which indicate the percentiles for the expected regulating price calculated by the model, describing the probability for a regulating price given a regulating volume.

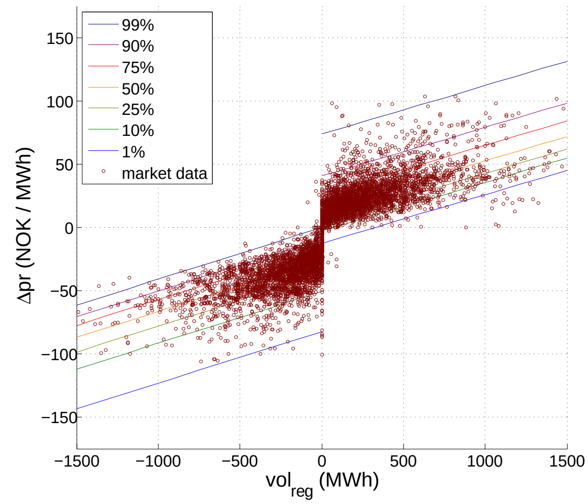


Fig. 3. The resulting statistical model, southern Norway 2007

The corresponding model parameters for the years 2003 to 2007 for southern Norway can be found in the Table I. The parameters are quite stable during these years. With the exception of 2006 κ_{up} is lower than κ_{down}, i.e. the influence of the regulating volume is higher in the case of downward regulation than in upward regulation. This indicates that upward regulation is cheaper to provide than downward regulation in southern Norway. The opposite effect in 2006 may be explained by the fact that this was a very dry year, increasing the cost of hydro generation. Also the comparison of the constant parameters of the linear model η_{up} and η_{down} indicate that provision of upward regulation is normally cheaper than downward regulation.

In this paper the long term modelling is done for each complete year. It is also possible to estimate the model parameters for other periods, whereby an appropriate number of hours is necessary to give reasonable results. Shorter time periods, for example three months can give the ability to investigate differences in the seasonal behaviour of the regulating market.

It should be noted that this model is designed to describe the normal behaviour of the regulating market. For extreme events a separate consideration should be done as they can have a large effect on profits or losses.

B. Generating short time scenarios

The long term model provides a statistical description of the whole market. However in order to use (2) as a model for generating regulating price scenarios, an additional method

TABLE I
MODEL PARAMETERS FOR SOUTHERN NORWAY IN 2003-2007

	2003	2004	2005	2006	2007
η_{up}	21.3178	11.1673	15.7429	11.6234	14.3344
κ_{up}	0.0248	0.0237	0.0287	0.0481	0.0384
μ_{up}	6.8624	3.9183	4.5540	6.6039	5.2576
σ_{up}	18.6553	13.2084	11.5373	19.0644	14.0912
p_{no}	0.2619	0.1262	0.1800	0.2188	0.1658
μ_{no}	6.7531	4.3791	8.6721	6.1136	9.9284
σ^2_{no}	29.7183	17.0842	29.7241	23.9360	20.2527
η_{down}	-35.8236	-22.2920	-24.7615	-34.4435	-27.0698
κ_{down}	0.0567	0.0311	0.0346	0.0360	0.0408
μ_{down}	8.5286	4.6718	5.3610	7.6164	6.4320
σ_{down}	20.0469	14.2187	15.6815	18.3343	13.3973

is needed to determine the regulating volume and regulating states. Beside a high dependence of Δpr on vol_{reg} there is an inherent time dependence of regulating prices. This inertia can be explained by the forecast errors for wind, the temperature or the general system state. Therefore it is necessary to include the time dependent behaviour due to the auto-correlation of Δpr. An example of the auto-correlation between subsequent hours during one year, here 2007, can be found in Fig.4. Considering the time dependence of the regulating prices, the perspective on the regulating market is changed from a general statistical model with the time scale of a year to a model which produces scenarios of regulating prices in a short term, i.e. for a period of two days.

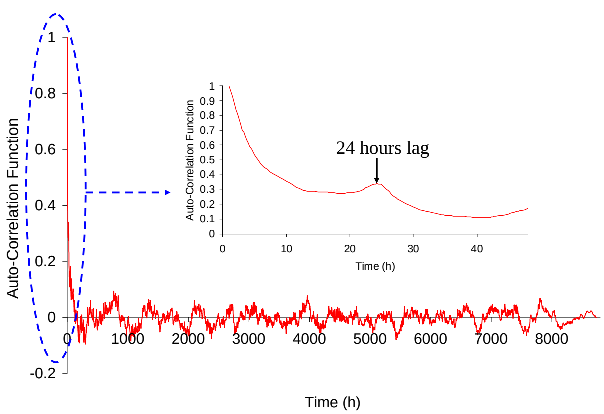


Fig. 4. Auto-correlation of regulating price difference

As can be seen from Fig.4, not only each successive Δpr is correlated but there is also a correlation between each 24 hours. Taking this into account, an appropriate stochastic process modelling time series should consider data for successive hours and successive days. The outcome of the model is the forecasted regulating state for the next time period. Furthermore an estimation of regulating volumes has to be done. Considering the forecasted regulating states and the estimated regulating volumes, different regulating price scenarios can be generated, using the model in the previous section.

1) Regulating State Determination: The focus of this modelling approach is to determine the future regulating states of the system based on the auto-correlation in the time series describing the regulating state during a certain period. Thus the model is based on time series processes. The process used is a so called SARIMA process which models hourly and daily dependence as well as a stochastic behaviour. SARIMA is an established model used to forecast load [5] and day-ahead spot market prices [6], also used to model real-time balancing power market prices [3]. SARIMA is a linear model for forecasting seasonal time series. It includes two major sets of parameter; regular parameter i.e. (p, d, q) and seasonal[1] parameters i.e. (P, D, Q). d is the number of regular differences and D is the number of seasonal differences. p, P, q and Q are the order of regular and seasonal auto-regressive (AR) and moving-average (MA) polynomials respectively. A SARIMA model can account for a temporal dependence in several ways. First, not only the absolute value but also the hourly difference can be used in order to make the process stationary (time series difference of the order d). Second, the time dependence of the stationary process is modelled by including p AR and q MA terms. For a cyclical time series, these steps can be repeated according to the period of the cycle, whether daily, quarterly, monthly or any other time interval parameters are designated as (P, D, Q). A general model of a SARIMA$(p, d, q) \times (P, D, Q)_S$ process is defined as (6).

$$\Phi\left(B^S\right)\varphi\left(B\right)\nabla_S^D\nabla^d X_t = \Theta\left(B^S\right)\theta\left(B\right)Z_t \qquad (6)$$

Where $\nabla_s^D = (1 - B^s)$ and $\nabla^d = (1 - B)$.

$\Phi\left(B^s\right)$, $\varphi\left(B\right)$, $\Theta\left(B^S\right)$ and $\theta\left(B\right)$ are polynomials expressed as follows:

$$\Phi\left(B^S\right) = 1 - \Phi_1 B^S - \Phi_2 B^{2S} - \ldots - \Phi_P B^{PS} \qquad (7)$$

$$\Theta\left(B^S\right) = 1 - \Theta_1 B^S - \Theta_2 B^{2S} - \ldots - \Theta_Q B^{QS} \qquad (8)$$

and,

$$\phi\left(B\right) = 1 - \varphi_1 B - \varphi_2 B^2 - \ldots - \varphi_p B^p \qquad (9)$$

$$\theta\left(B\right) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q \qquad (10)$$

In addition, Z_t is a white noise sequence, $Z_t \sim WN\left(0, \sigma^2_{WN}\right)$.

The maximum likelihood method using a bias-corrected version of the information criterion of Akaike is used to estimate the parameters [7]. The "System Identification Toolbox" in MATLAB is used to implement the model.

The recorded time series of regulating prices in a previous week is used for identifying the process and fitting parameters. Regulating states indicated by the relation between forecasted time series and the spot price are determined for 48 hours ahead. The hourly regulating prices for the week from the 04.05.2007 to the 10.05.2007 is selected as the observed data and the regulating states are forecasted for the next 48 hours. The time series is modelled with a SARIMA$(1, 1, 2) \times (1, 1, 2)_{24}$ process with the following polynomials:

[1]The regular length in this model is one hour and the seasonal length is 24 hours describing the day to day dependence.

$$\begin{aligned}
\varphi(B) &= 1 - 0.2027 \cdot B \\
\Phi(B) &= 1 + 0.08438 \cdot B^{24} \\
\theta(B) &= 1 - 0.2552 \cdot B + 0.2826 \cdot B^2 \\
\Theta(B) &= 1 - 0.09513 \cdot B^{24} - 0.28 \cdot B^{48} \\
,\text{with } Z_t &\sim (0, 218.7481)
\end{aligned}$$

The sample auto-correlation (ACF) and the partial auto-correlation[2] (PACF) functions for the generated model residuals [7] were calculated. The results can be studied in Fig.5 where the horizontal lines correspond to the interval $\pm\frac{1.96}{\sqrt{n}}$ with n being the sample size. Based on the criterion of an acceptable fitting suggested in [7], if the process includes a white noise sequence, 5% of the residuals can be expected to fall outside the interval. As shown in Fig.5, less than 4.92% of the residuals for ACF and PACF fall outside the interval, which indicates a good model fit.

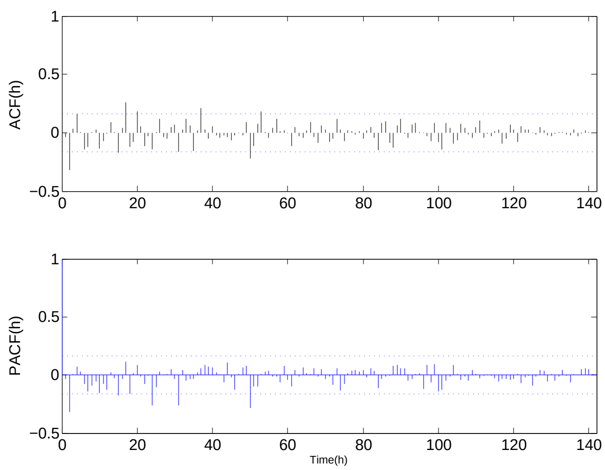


Fig. 5. ACF and PACF calculated for the residuals. The dotted horizontal line represents the interval $\pm\frac{1.96}{\sqrt{n}}$

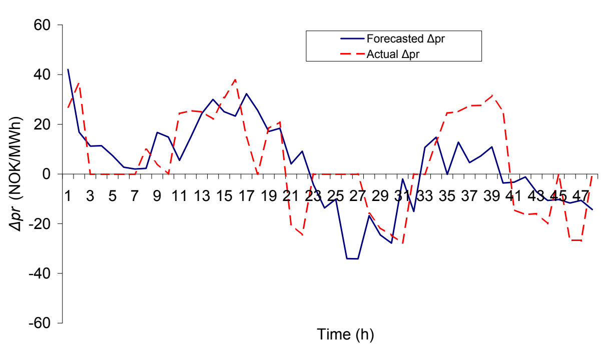


Fig. 6. Forecasted Δpr versus actual Δpr

The model is used for forecasting the regulating states within the next 48 hours. The model's outputs are shown in Fig.6. In order to determine the regulating state, the forecasted regulating prices are compared with the actual spot prices. The points lying above the horizontal axis represents the upward regulating states and below the downward regulating states. The white noise sequence prevents the forecasted Δpr from remaining at zero level which would be the no regulating state. Hence a hypothetic band is introduced around the spot prices to define the no regulating state. The width of the band is determined in such a way that the percentage of points lying inside the band is according to the percentage of no regulating states in the observed data i.e. the recorded data during the previous week. The upper and lower limits of the band are assumed to be l_d and l_u where l_d is lower and l_u is higher than spot price. l_d and l_u are considered to lie symmetrically around the spot price. If the forecasted price is above l_u the system is in the upward regulation state and if it is below l_d the system is in the downward regulation state. Between l_d and l_u the system is in the no regulation state. Fig.7 depicts how each state is determined.

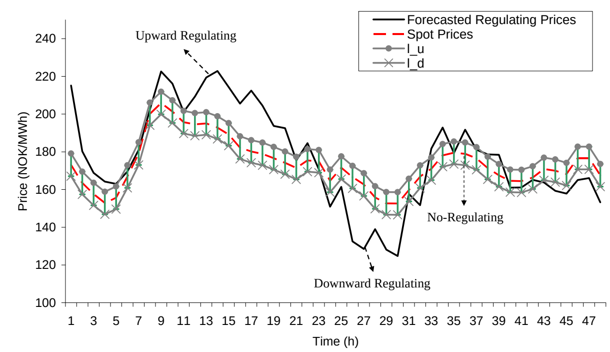


Fig. 7. Regulating state determination

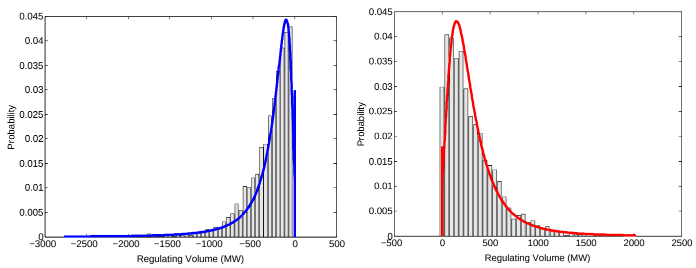
It should be mentioned that the regulating volumes can be determined in the state determination step inherently. However in order to study the effect of changing regulating volumes on regulating prices, it is necessary to treat the volume as a variable. Thus the only important parameter considered in this step is the auto-correlation of regulating state occurrence.

2) Statistical Properties of Regulating Volume: Given the regulating state, based on the statistical properties of the regulating volume, volume scenarios can be generated. Looking on the recorded Norwegian balancing market data, a statistical model of the regulating volume is defined. Since the regulating states are determined for upward and downward regulating separately, the statistical distribution should be determined for each state separately. Considering the asymmetric property of the distribution of the regulating volume where one tail is relatively wide while the other one behaves quite like a normal distribution the extreme value theory is able to capture these features. More details can be found in the appendix.

Given the recorded regulating volume, maximum likelihood estimates the parameters for a Generalized Extreme Value (GEV) distribution. Fig.8 shows the fitted distribution and observed values for the regulating volume. The estimated parameters are as follows:

[2]PACF at the given lag removes the effect of shorter lag autocorrelation from the correlation estimate at longer lags.

$$\begin{aligned} \mu_{up}^{vol} &= 187.7703 & \mu_{down}^{vol} &= -147.3467 \\ \sigma_{up}^{vol} &= 157.0335 & \sigma_{down}^{vol} &= -134.7605 \\ k_{up}^{vol} &= 0.2681 & k_{down}^{vol} &= 0.4221 \end{aligned}$$



(a) downward (b) upward

Fig. 8. Fitted GEV for downward and upward regulating volume

It should be noted that there are a few special hours within the year which are identified as up- or downward regulating due to difference between regulating price and spot price but where no regulating volume is used in the area (i.e. southern Norway). This can be explained by the impact of the other areas as mentioned in part III-A. The probability of these cases has been taken into account with the spike at zero shown in Fig.8.

3) Scenario Generation: The regulating state and the estimated regulating volume are used as the input for the previous statistical model, determining the difference between the regulating and the spot price. The results of this modelling approach are regulating price scenarios, indicating different percentiles of the regulating volume, depicted in Fig.9. This figure shows that there is no considerable difference between the percentiles of the regulating volume scenarios. This can be explained by the characteristics of the regulating volume in southern Norway, cf. Fig.8 which shows that the probability of high regulating volumes is very low. Also Fig.3 shows small price difference for low volumes.

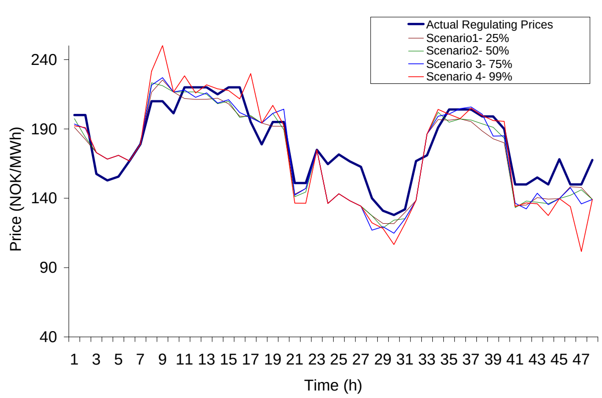


Fig. 9. Regulating price scenarios

IV. Conclusion

Interconnections between separate control areas are normally used for economic exchange and for mutual support in the case of outages. However, there is an increasing interest also to use such interconnections for the exchange of regulating power. This is especially the case for the HVDC interconnection between Norway and the Netherlands.

In order to study the effect of the exchange of regulating power on the Norwegian market, a model is proposed, which is split into a long-term and a short-term part. The long-term model consists of a deterministic and a stochastic part with error terms that are based on extreme value theory. The long term model describes the price-volume dependence in the regulating market where the regulating volume is an input to the model. The second step considers the short-term situation, taking into account the auto-correlation in regulating market prices. The short term model is based on a SARIMA process, and computes a forecast of future regulating states. With a statistical description of the regulating volume, scenarios are generated that are the input to the long-term model resulting in regulating price scenarios.

The aim of the models is to enable exploration of the impact of the cross border trading of regulating resources. This is possible as the regulating volume is the input to the long term model. As a result of this the generated regulating volumes can be changed according to the exchange of regulating resources in integrated regulating markets. Even with a change in the interaction with other markets or areas, the statistical model can still be a good description for the market, as long as the parameters describing the characteristics of the supply side of the market stay constant. Thus the model is based on historical market data and intended to be used for the investigation and evaluating future market behaviour.

Further work will focus on the implementation of areas within central continental Europe connected to southern Norway through HVDC cables. A design framework is needed for an efficient regulating power cross border trading. The proposed model is currently expanded to include the impact of trading with other areas and the harmonisation of balance regulation in the Nordic countries in 2009.

Appendix

The distribution functions used in this paper are consecutively described in more detail. $C(p)$ is a discrete 0,1-distribution defined by

$$C(p) \in [0;1] \tag{11}$$

$$P(C(p)=1) = p \tag{12}$$

$N(\mu, \sigma^2)$ is a normal distribution with the mean value μ and the standard deviation σ^2. The probability density function is given as:

$$f_N(x|\mu, \sigma^2) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{13}$$

In order to be able to capture asymmetric as well as "fat-tail" distribution functions the extreme value theory is applied. The Generalized Extreme Value distribution $GEV(k, \mu, \sigma)$ is a continuous probability distribution, developed within the extreme value theory. It combines the Gumbel, Fréchet and Weibull distribution also known as type I, II and III extreme value distribution respectively. The probability density function is given in (14).

$$f_{GEV}\left(x|k,\mu,\sigma\right)=\frac{1}{\sigma}\cdot e^{-\left(1+k\cdot\frac{x-\mu}{\sigma}\right)^{-\frac{1}{k}}\cdot\left(1+k\cdot\frac{x-\mu}{\sigma}\right)^{-1-\frac{1}{k}}} \quad (14)$$

The different types are defined by the shape parameter k, with type I for $k \rightarrow 0$, type II for $k > 0$ and type III for $k < 0$. Type I is also called Extreme Value distribution. $EV\left(\mu,\sigma\right)$ is a distribution with the location parameter μ and the scale parameter σ. Its probability density function is given as follows:

$$f_{EV}\left(x|\mu,\sigma\right)=\frac{1}{\sigma}\cdot e^{-\frac{x-\mu}{\sigma}-e^{-\frac{x-\mu}{\sigma}}} \quad (15)$$

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