Upgridding by Amalgamation: Flow-Adapted Grids for Multiscale Simulations

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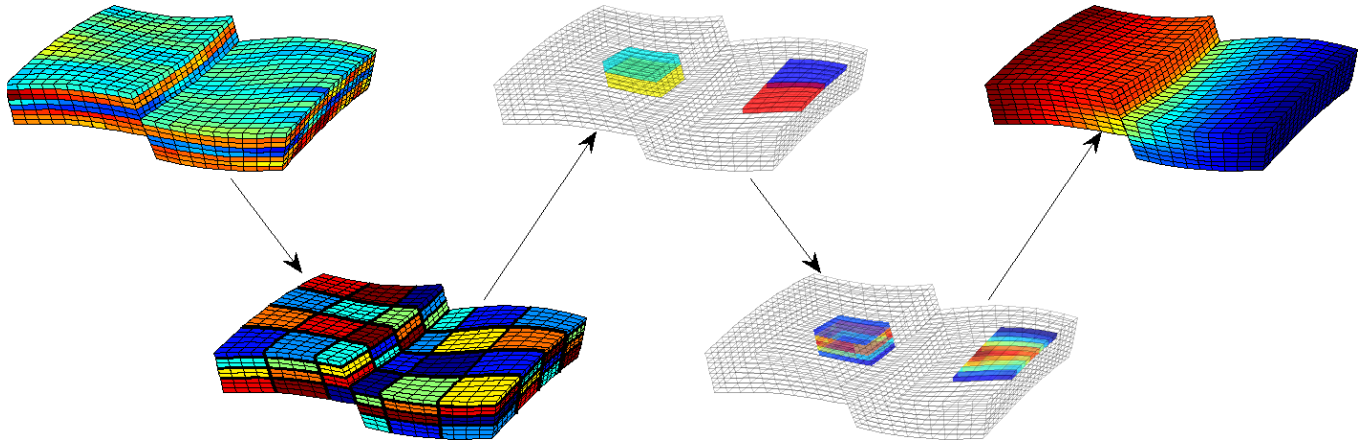

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Long Beach, CA, March 21–24, 2011

What is multiscale simulation?

Generally:

Methods that incorporate fine-scale information into a set of coarse scale equations in a way which is consistent with the local property of the differential operator

Herein:

Multiscale pressure solver (upscaling + downscaling in one step)

$$\nabla \cdot \vec{v} = q, \qquad \vec{v} = -\lambda(S)\mathbf{K}\nabla p$$

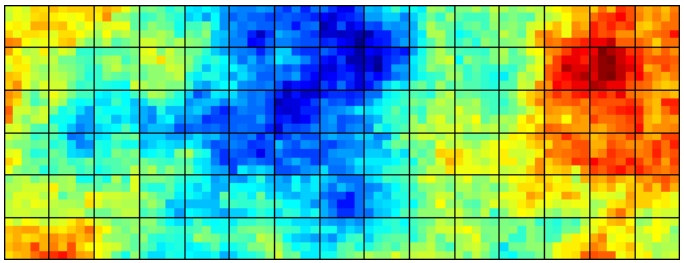
\+ Transport solver (on fine, intermediate, or coarse grid)

$$\phi \frac{\partial S}{\partial t} + \nabla\Big(\vec{v} f(S)\Big) = q$$

= Multiscale simulation of models with higher detail

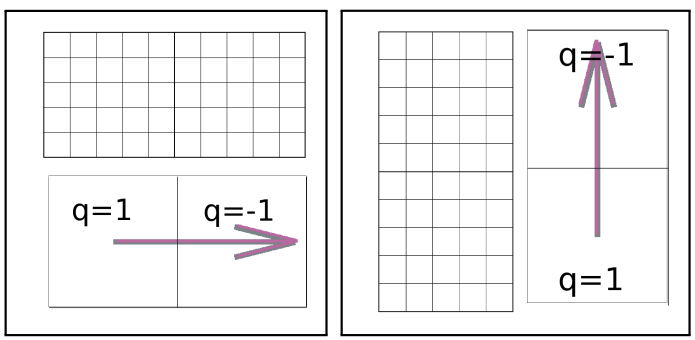
What is multiscale simulation?

Coarse partitioning:



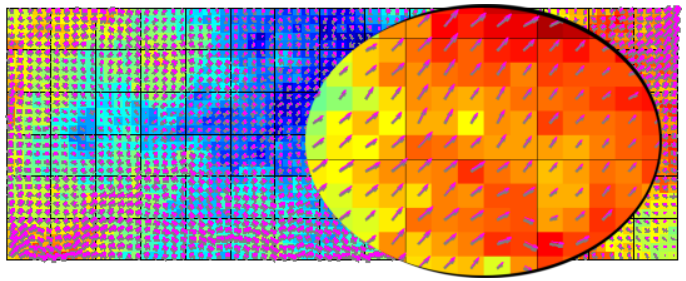
$\Downarrow$

Local flow problems:



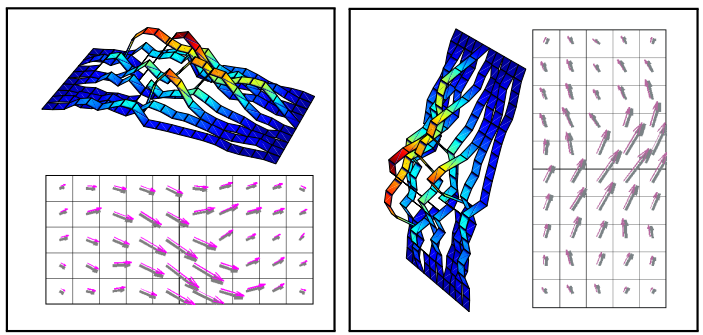
$\Rightarrow$

Flow field with subresolution:



$\Uparrow$

Flow solutions $\longrightarrow$ basis functions:



Flow-adapted coarse grids

Goal:

Given the ability to model velocity on geomodels and transport on coarse grids: Find a suitable coarse grid that best resolves fluid transport and minimizes loss of accuracy.

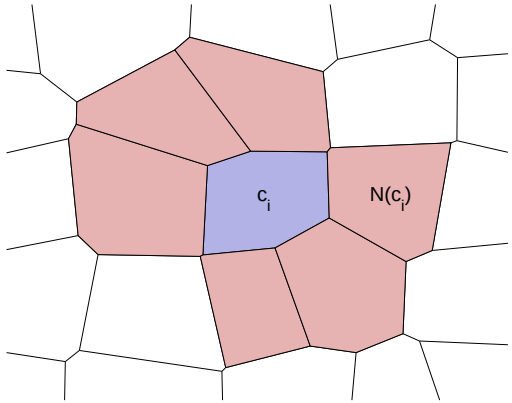
Formulated as the minimization of two measures:

1. the *projection error* between fine and coarse grid
2. the *evolution error* on the coarse grid

Flow-adapted coarsening

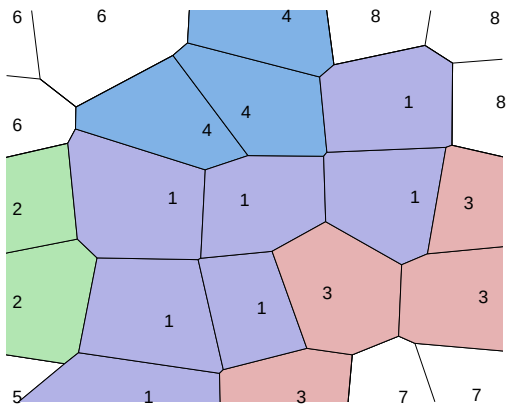
Assumptions:

- a matching polyhedral grid with n cells c_i,
- a mapping $\mathcal{N}(c)$ between cell c and its nearest neighbours,
- a set of flow indicators $I(c_i)$ in cell c_i



We seek a coarse grid that:

- adapts to the flow pattern predicted by indicator I,
- is formed by grouping cells into N blocks B_ℓ,
- is described by a partition vector p with n elements, in which element p_i assumes the value ℓ if cell c_i is member of block B_ℓ.



Coarsening principles

- Minimize heterogeneity of flow field inside each block

$$\min_{B_j}\Big(\sum_{p_i=j}|I_1(c_i)-I_1(B_j)|^p\,|c_i|\Big)^{\frac{1}{p}},\qquad 1\le p\le\infty,$$

- Equilibrate indicator values over grid blocks

$$\min\Big(\sum_{j=1}^{N}|I_2(B_j)-\bar{I}_2(\Omega)|^p\,|B_j|\Big)^{\frac{1}{p}},\qquad 1\le p\le\infty,$$

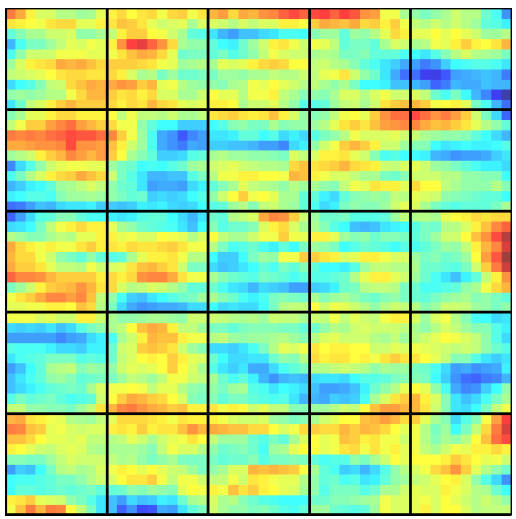
- Keep block sizes within prescribed lower and upper bounds

Amalgamation algorithm

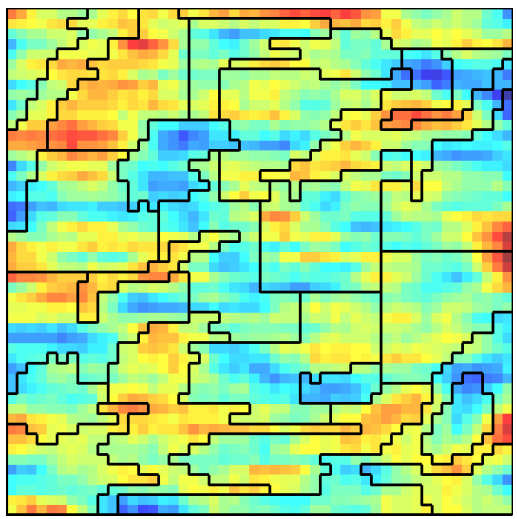
Amalgamation of cells:

- Difficult to formulate a practical and well-posed minimization problem for optimal coarsening $\longrightarrow$ ad hoc algorithms
- Coarsening process steered by a set of admissible and feasible amalgamation directions

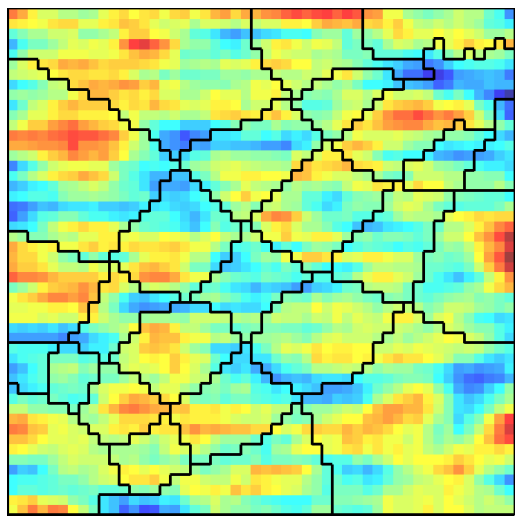
50×50 **lognormal permeability:**



regular: 25 blocks



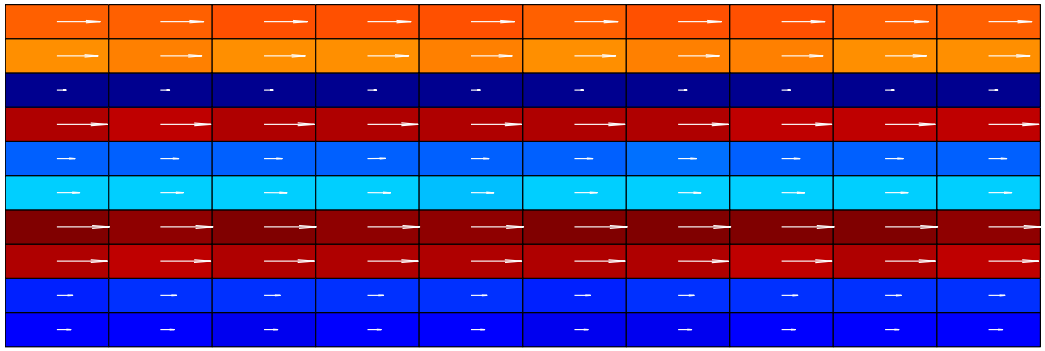
flow magnitude: 26 blocks



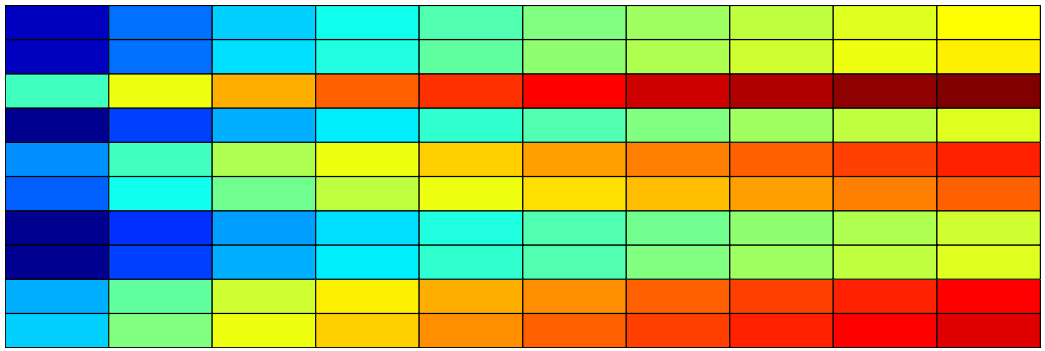
isocontours [p]: 26 blocks

Motiviation: layered reservoir

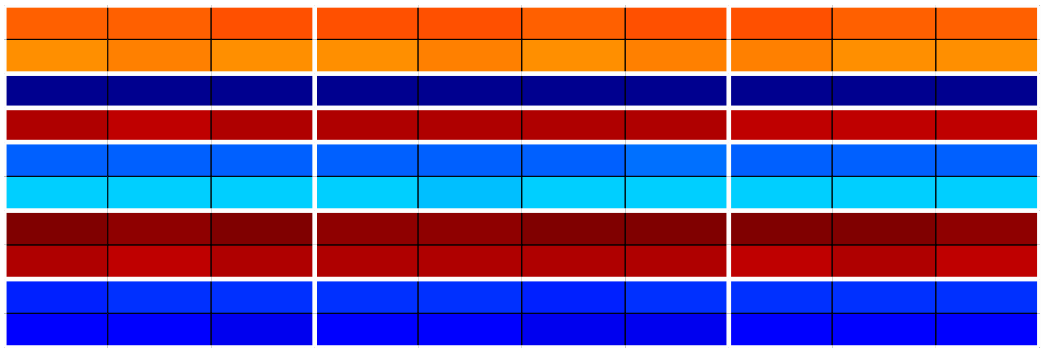
Permeability and velocity



Time−of−flight



Partition



Heuristic minimization: algorithmic components

Formulated using a set of:

sources that create a partition vector based upon grid topology, geometry, flow-based indicator functions, error estimates, or expert knowledge supplied by the user, thereby introducing the feasible amalgamation directions

`filters` that take a set of partition vectors as input and create a new partition as output, by

- combining/intersecting different partitions
- performing sanity checks, ensuring connected partitions according to admissible directions, etc
- modifying partition by merging small blocks or splitting large blocks

Heuristic minimization: algorithmic components

Partition:

- prescribed topology or predefined shapes
- segmentation of cells c_i into bins $\tilde{B}_\ell$

$$c_i \subset \tilde{B}_\ell \text{ if } I(c_i) \in [\ell, \ell + 1).$$

assuming indicator function I scaled to the interval $[1, M + 1]$

Intersection:

- intersect two or more partitions to produce a new partition
- split multiply connected blocks into sets of singly connected cells

Heuristic minimization: algorithmic components

Merging:

If block B violates the condition

$$I(B)\,|B| \geq \frac{N_L}{n}\,\bar{I}(\Omega)\,|\Omega|,$$

for a prescribed constant N_L, the block is merged with the neighbouring block B' that has the closest indicator value, i.e.,

$$B' = \operatorname{argmin}_{B'' \subset \mathcal{N}(B)} |I(B) - I(B'')|.$$

Refinement:

Refine blocks B that violate the condition

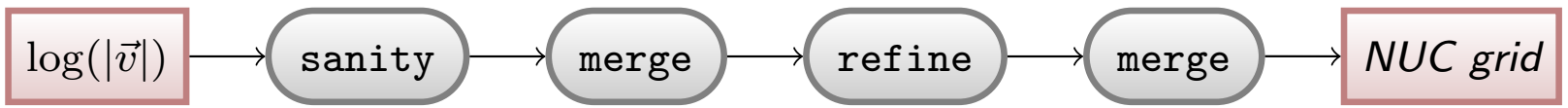
$$I(B)\,|B| \leq \frac{N_U}{n}\,\bar{I}(\Omega)\,|\Omega|,$$

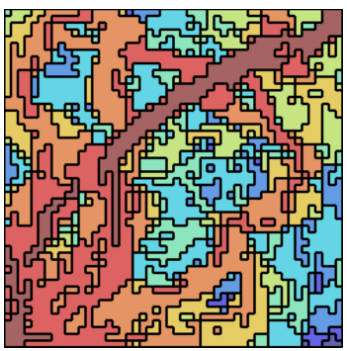
for a prescribed constant N_U

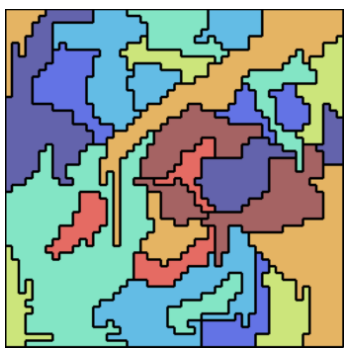
Example: non-uniform coarsening

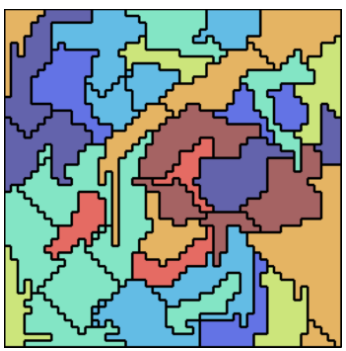
Aarnes, Efendiev & Hauge (2007):

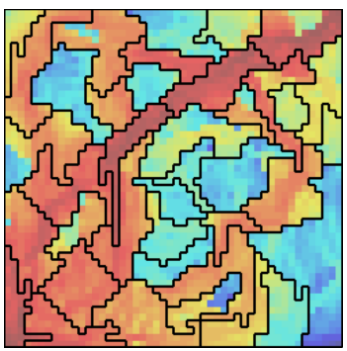
Use flow velocities to make a nonuniform grid in which each coarse block admits approximately the same total flow.



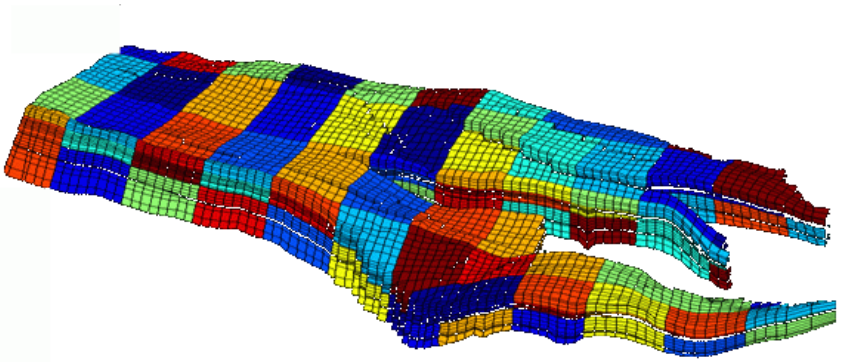

Partition: 304 blocks

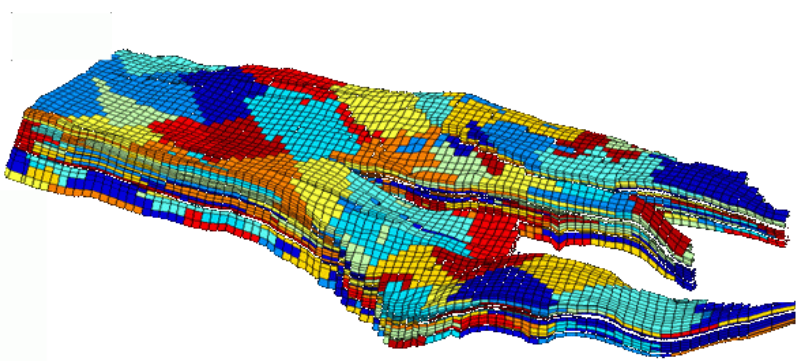

Merging: 29 blocks

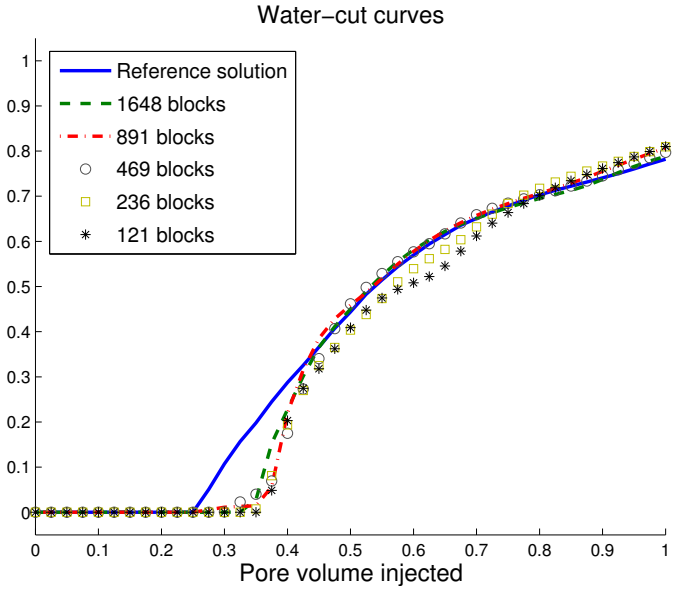

Refinement: 47 blocks

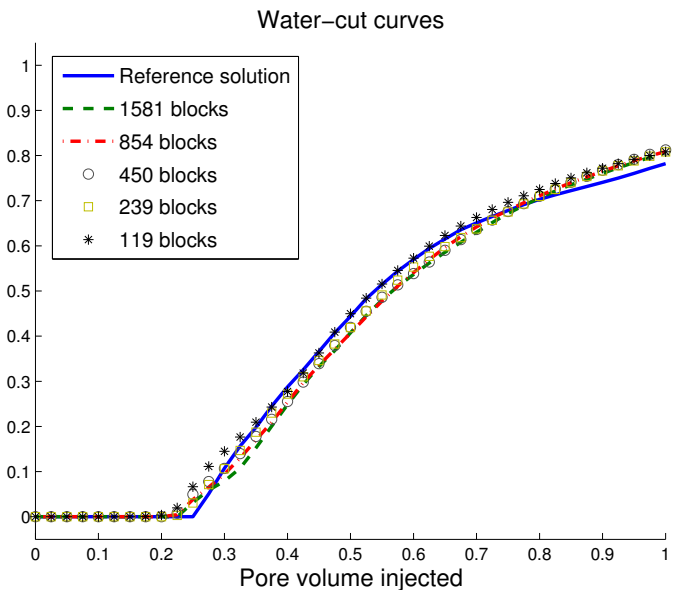

Merging: 39 blocks

Example: non-uniform coarsening

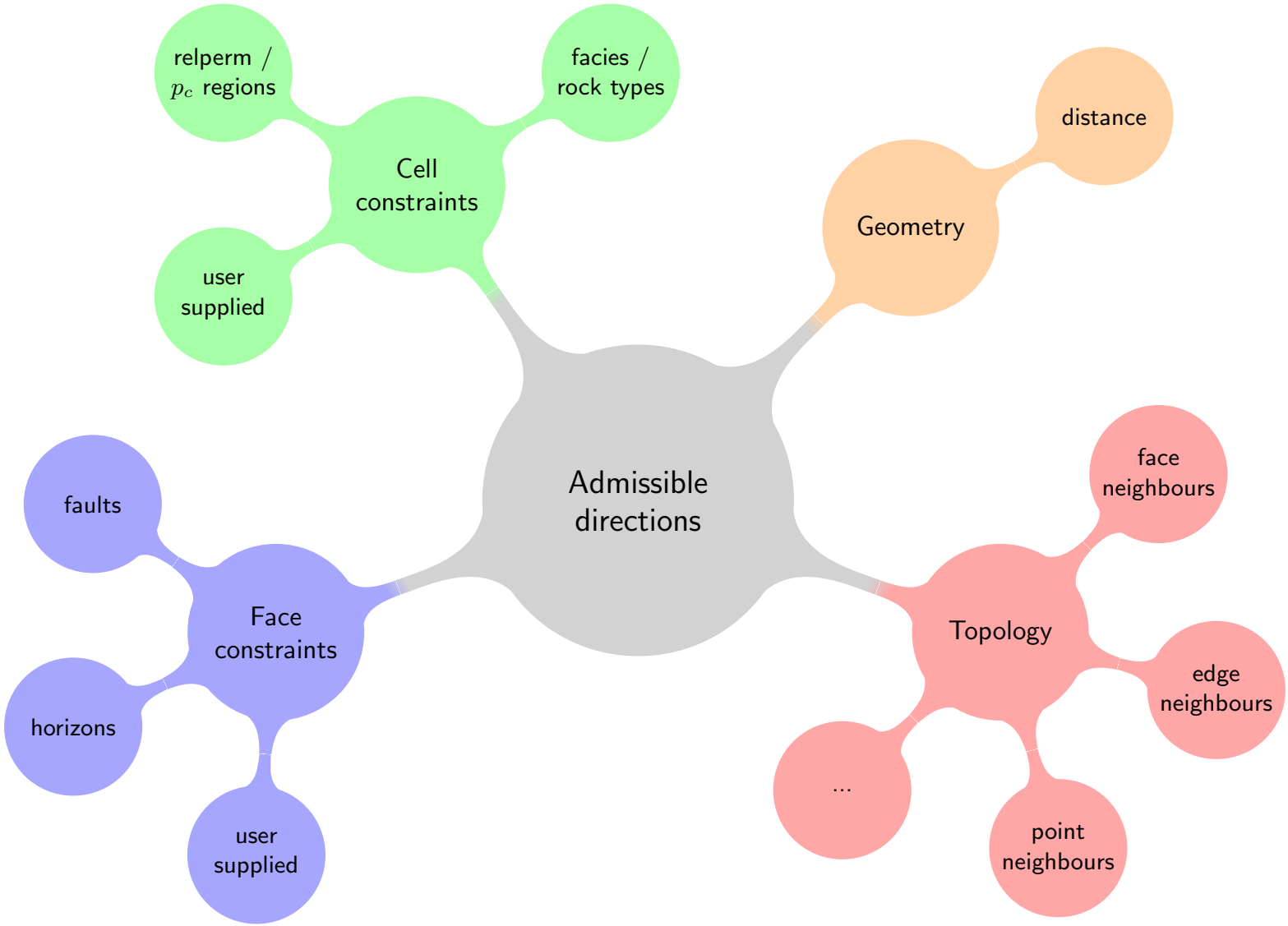




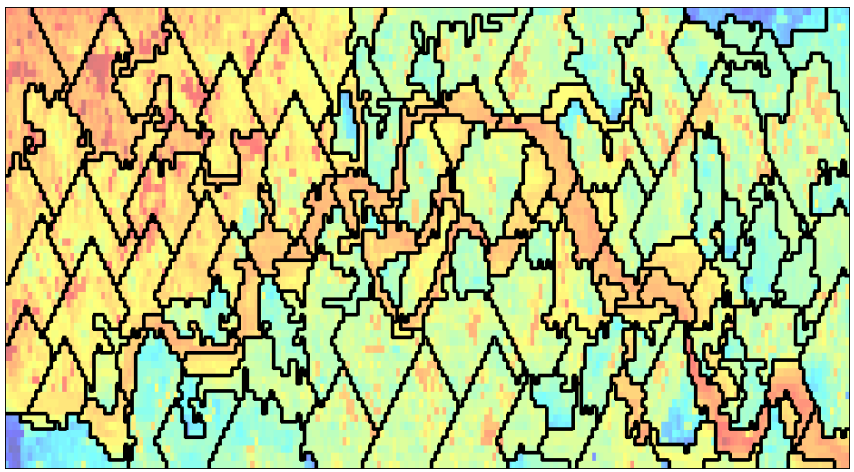


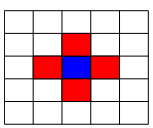


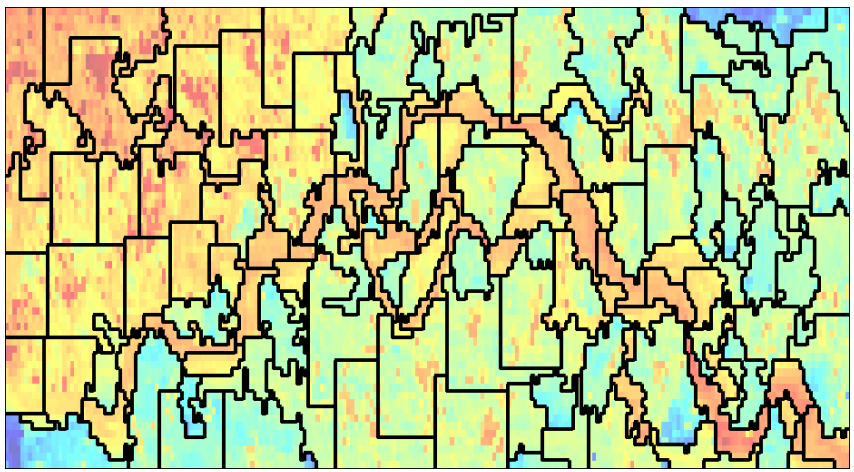
Amalgamation: admissible directions (neighbourship)

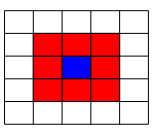


Amalgamation: extended neighbourship (topology)

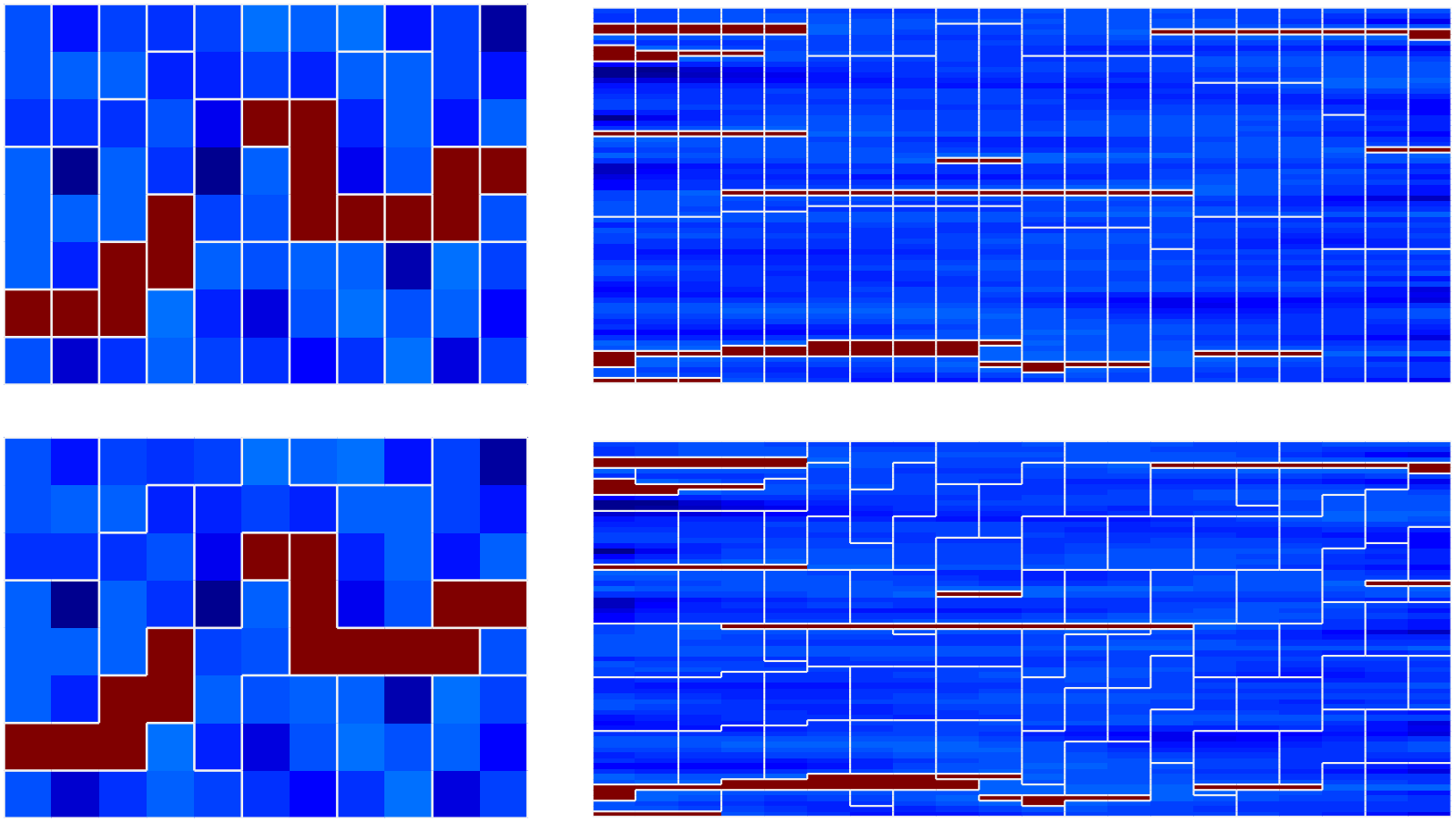


5-neighborhood 



9-neighborhood 

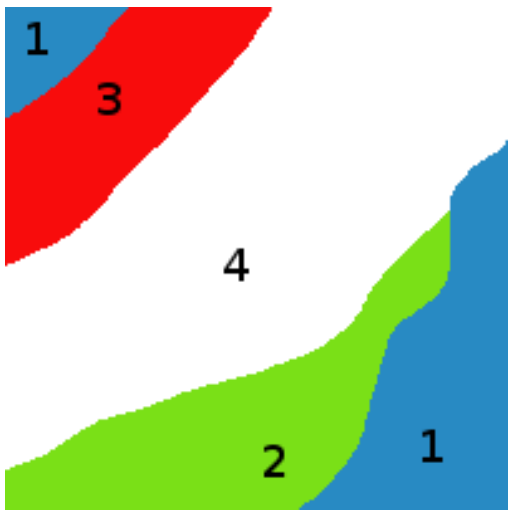
Amalgamation: restricted neighbourship (topology)



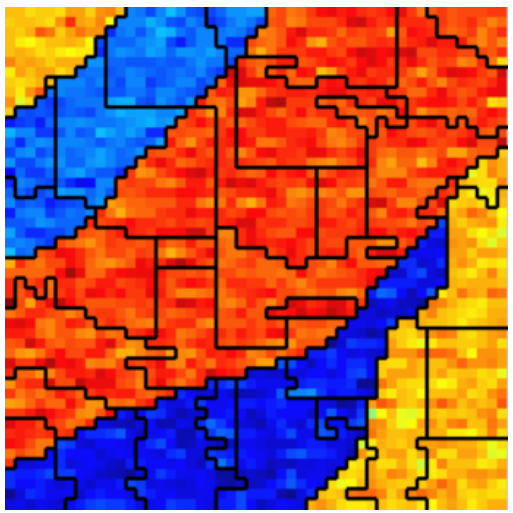
Upper row: $\mathcal{N}(c_{ij}) = \{c_{i,j\pm1}\}$

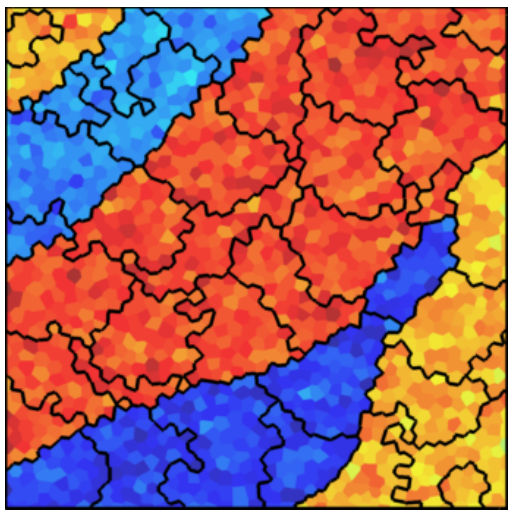
Lower row: $\mathcal{N}(c_{ij}) = \{c_{i,j\pm1}, c_{i\pm1,j}, c_{i\pm1,j\pm1}\}$

Amalgamation: restricted neighbourship (facies)



Facies distribution

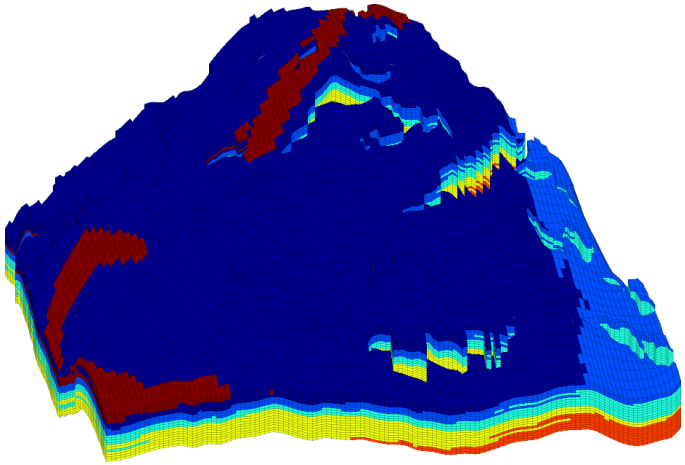

Cartesian


PEBI

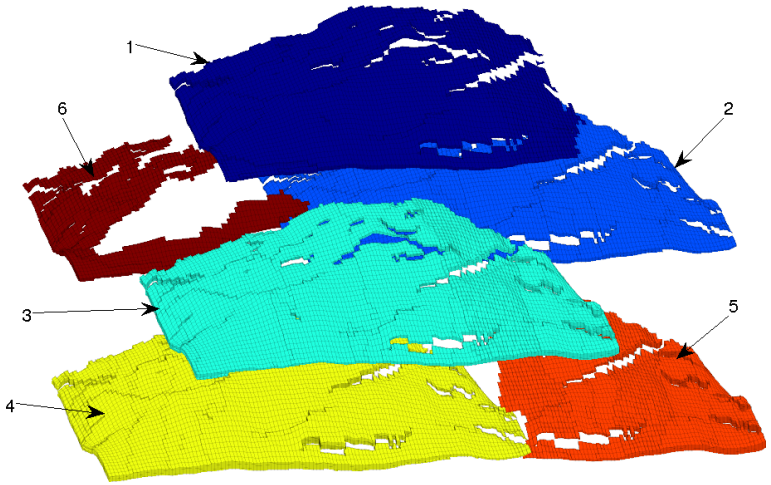
Constraining to facies / saturation regions:

- useful to preserve heterogeneity
- useful to avoid upscaling k_r and p_c curves

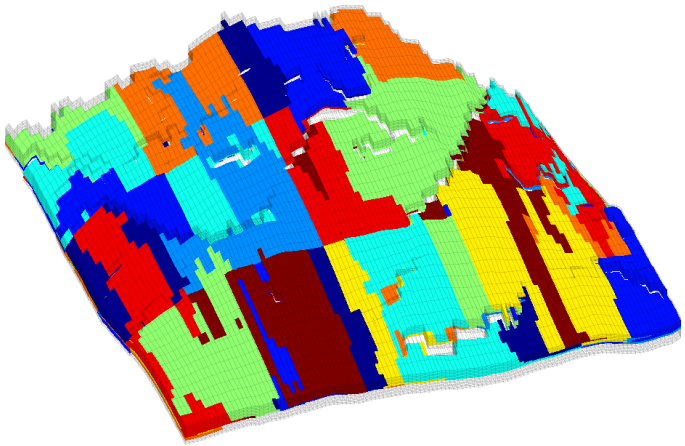
Amalgamation: restricted neighbourship (satnum)



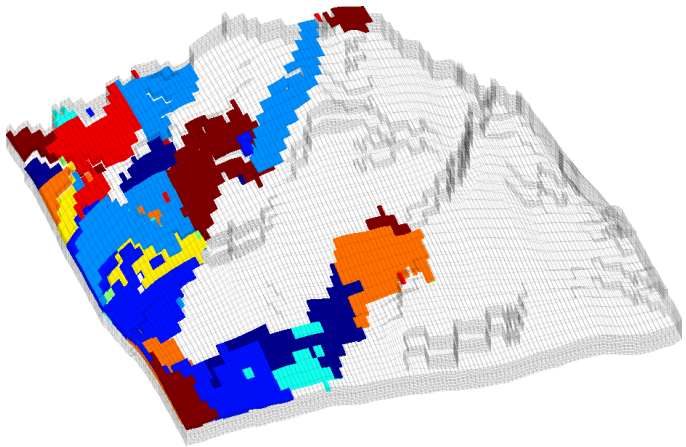
saturation regions



regions separated



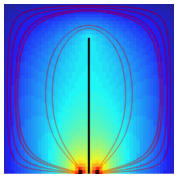
region # 3



region #6

Realization from SAIGUP study, coarsening within six different saturation regions

Amalgamation: restricted neighbourship (faults)

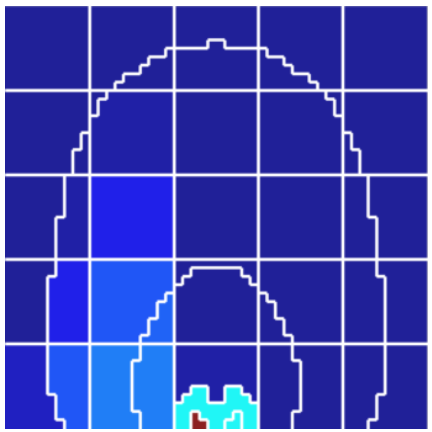


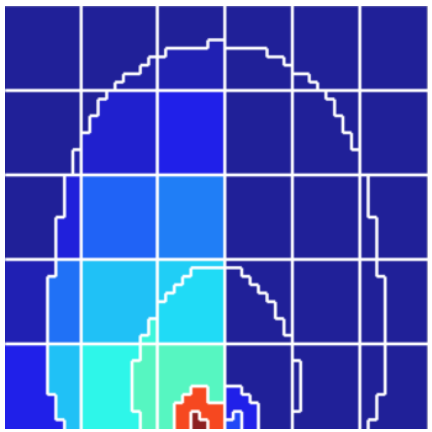
5×5 partition

6×5 partition

water-cut curves

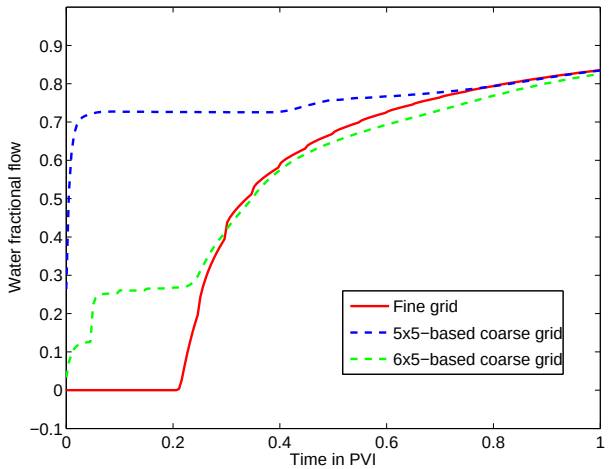
unconstrained



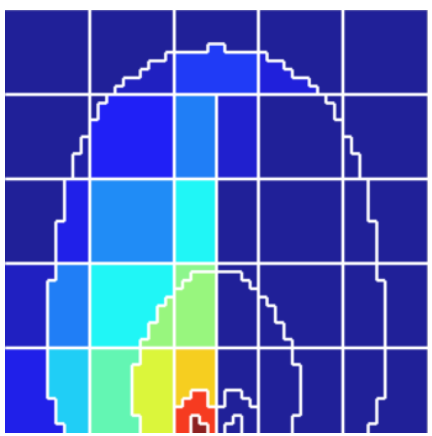


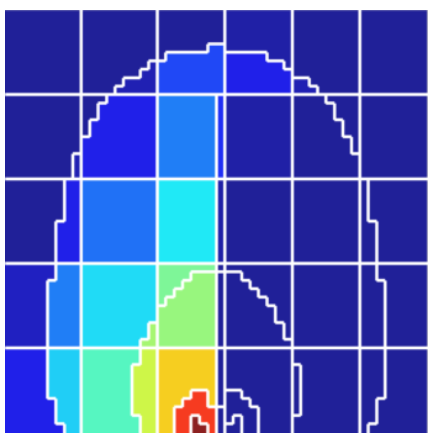
46 blocks

52 blocks



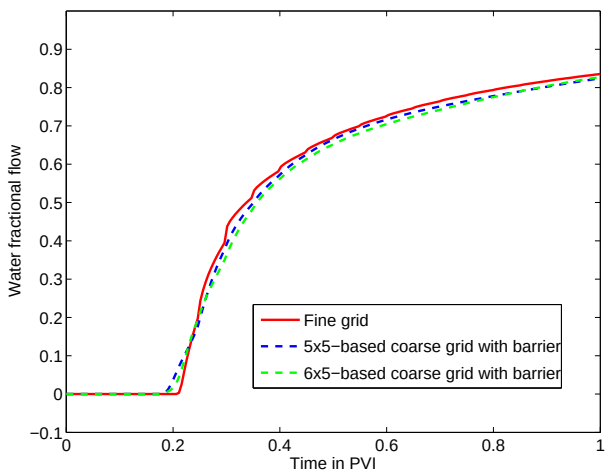
constrained



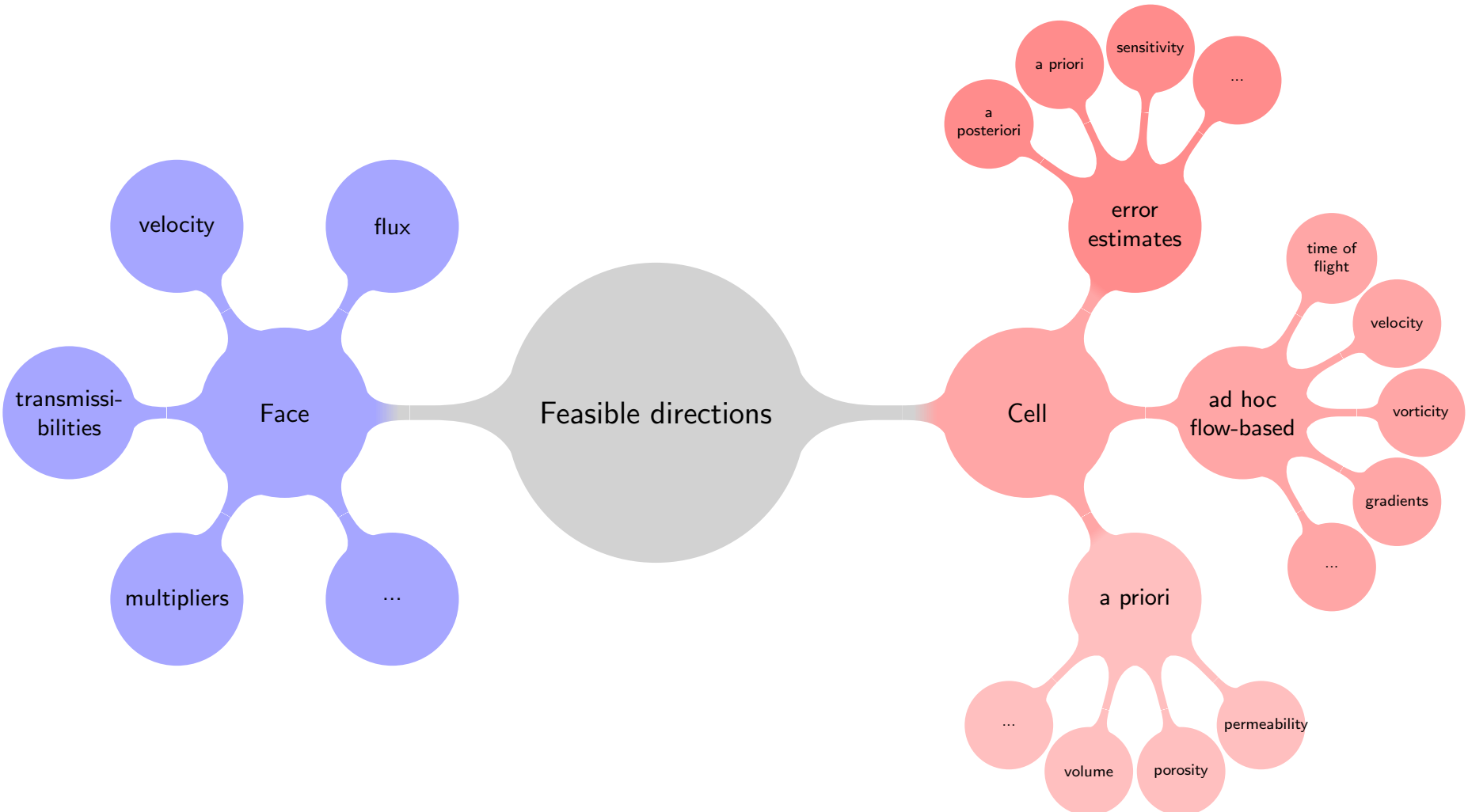


52 blocks

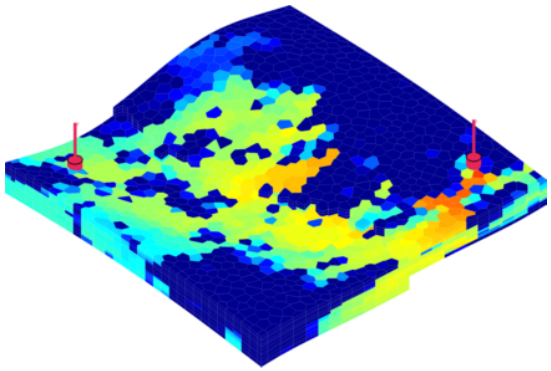
58 blocks



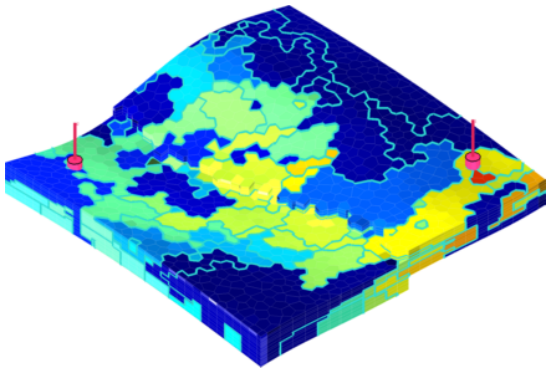
Amalgamation: feasible directions (indicators)



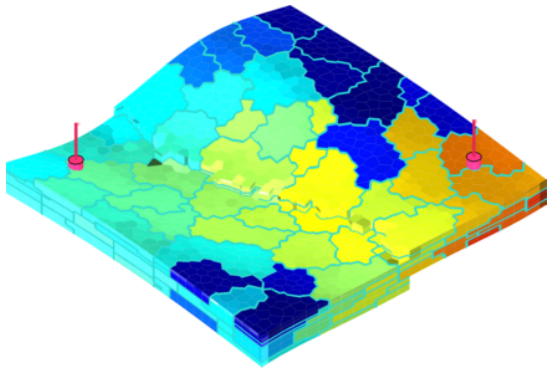
Example: flow-based indicators



reference solution
11 864 cells



time-of-flight grid
127 blocks



METIS grid
175 blocks

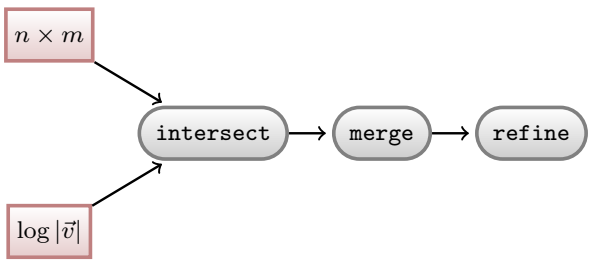
General observations:

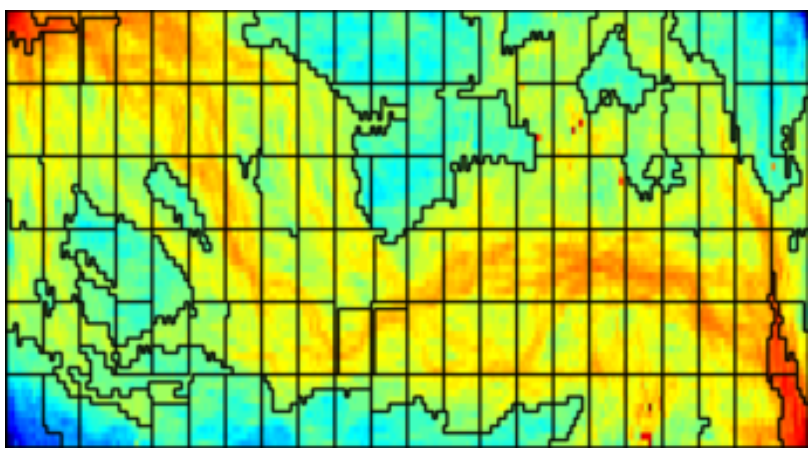
- Time-of-flight is typically a better indicator than velocity
- Velocity is a better indicator than vorticity
- Vorticity is a better indicator than permeability
- ...

However, for smooth heterogeneities, the indicators tend to overestimate the importance of flow.

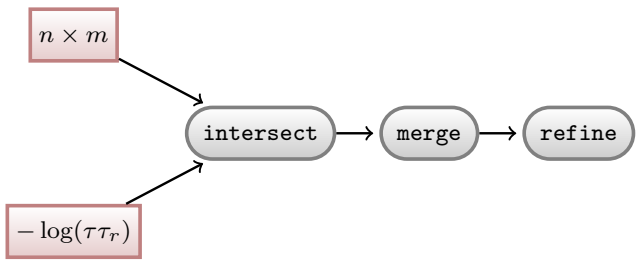
Example: hybrid methods

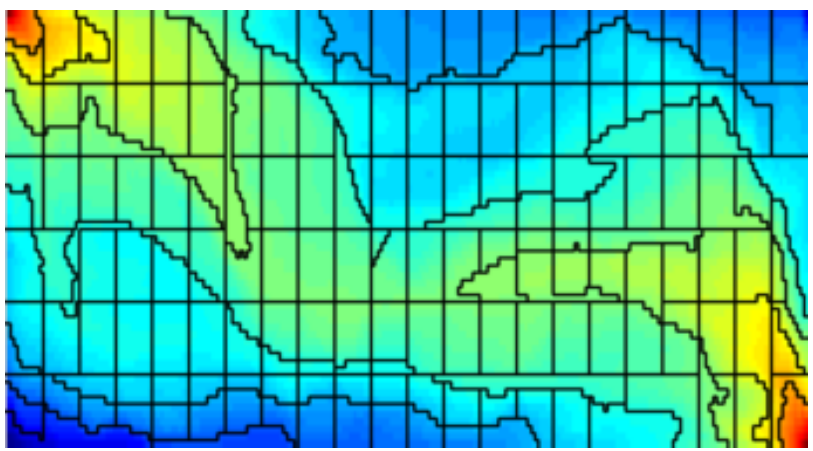
Velocity + Cartesian partition:





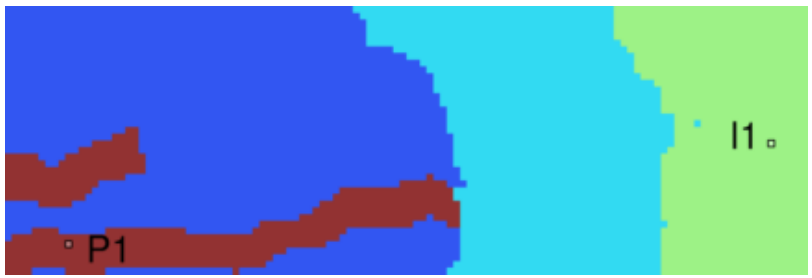
Time-of-flight + Cartesian partition:

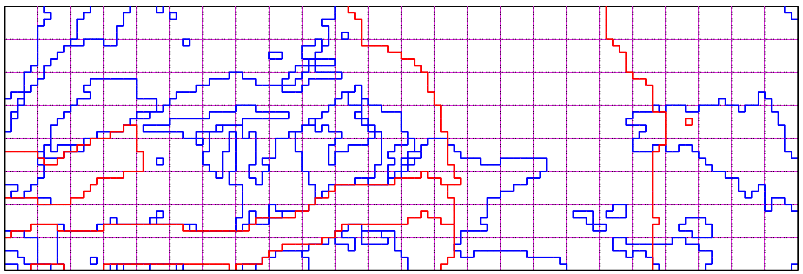


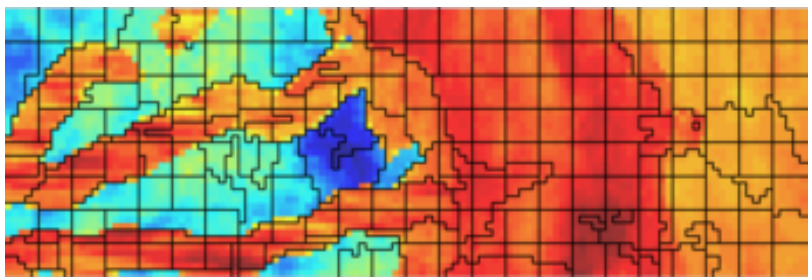


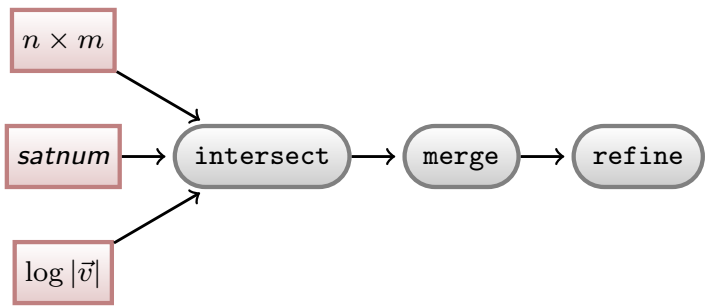
Example: hybrid methods

Satnum + velocity + Cartesian:









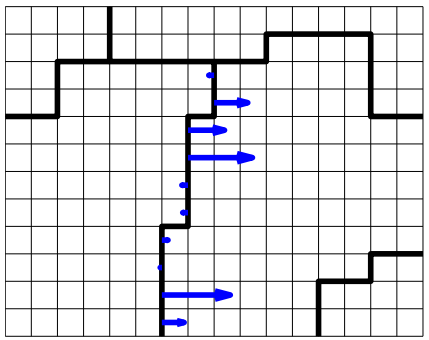
Flow-adapted coarsening: summary

- Developed a general and flexible framework
  - Heuristic algorithms: good rather than optimal grid
  - Algorithmic components: partition, intersection, merging, refinement
  - Key concepts: flow indicator, admissible and feasible directions
- Systematic way of generating fit-for-purpose grids
  - Several existing methods appear as special cases
- Inclusion of geological information and expert knowledge important
  - Facies, saturation regions, surfaces, faults, etc.
  - Predefined shapes and topologies

Coarse-grid discretisation

Bi-directional fluxes (upwind on fine scale):

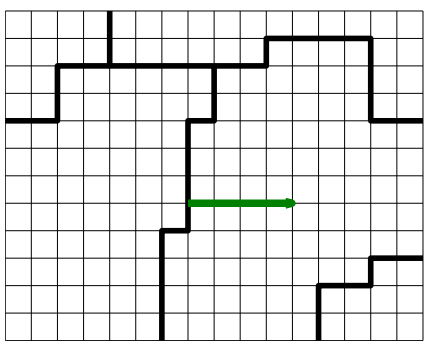
$$S_\ell^{n+1} = S_\ell^n - \frac{\Delta t}{\phi_\ell |B_\ell|}\Big[f(S_\ell^{n+1}) \sum_{\partial B_\ell} \max(v_{ij}, 0) - \sum_{k \neq \ell} \Big(f(S_k^{n+1}) \sum_{\Gamma_{k\ell}} \min(v_{ij}, 0) \Big) \Big].$$



This gives a centred scheme on the coarse scale

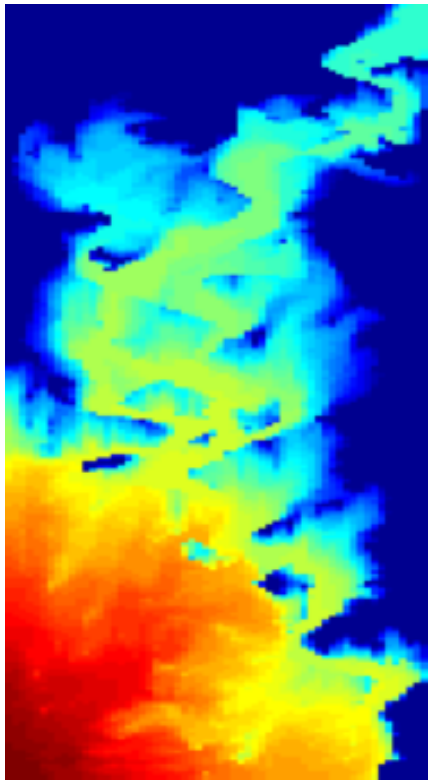
Net fluxes:

$$S_\ell^{n+1} = S_\ell^n - \frac{\Delta t}{\phi_\ell |B_\ell|} \sum_{k \neq \ell} \max\Big(f(S_\ell^{n+1}) \sum_{\Gamma_{k\ell}} v_{ij}, - f(S_k^{n+1}) \sum_{\Gamma_{k\ell}} v_{ij} \Big).$$

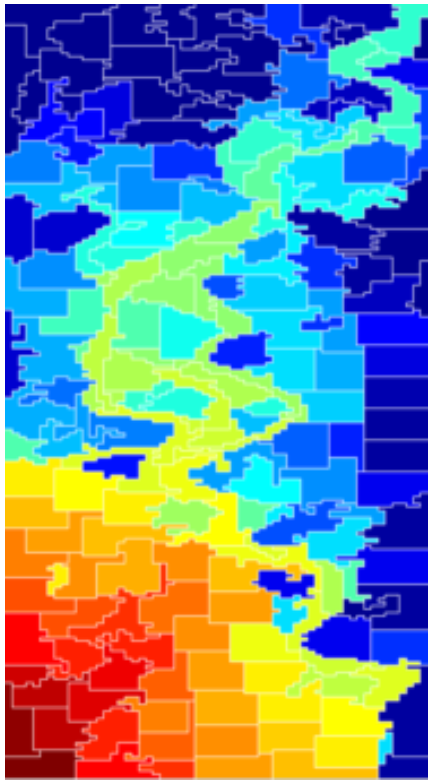


This gives an upwind scheme on the coarse scale

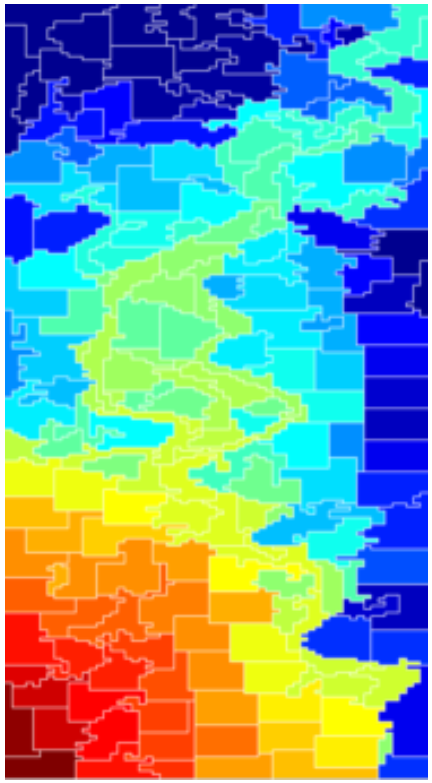
Coarse-grid discretisation: numerical diffusion



reference



net fluxes

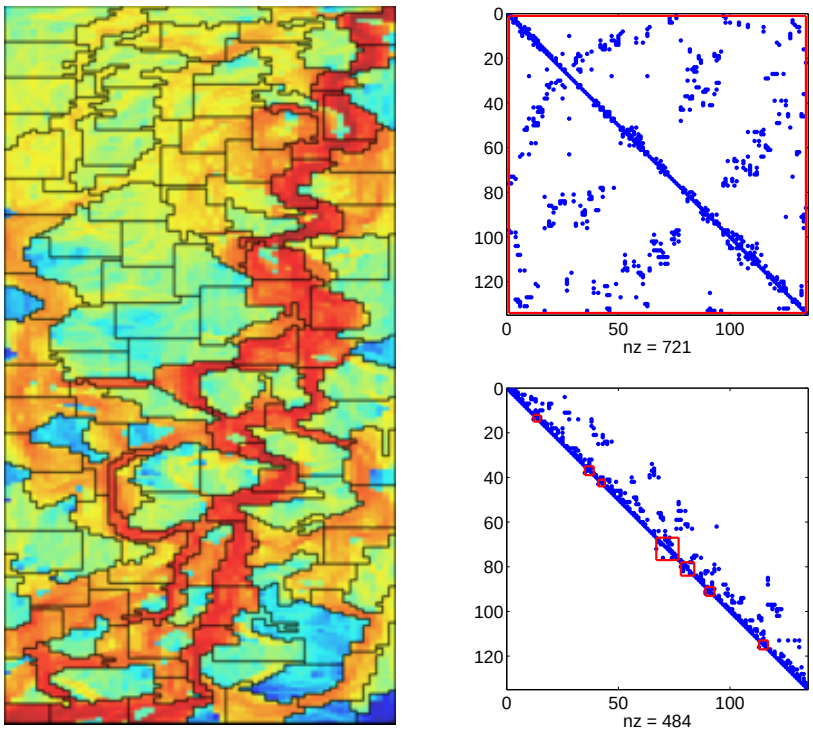


bi-directional fluxes

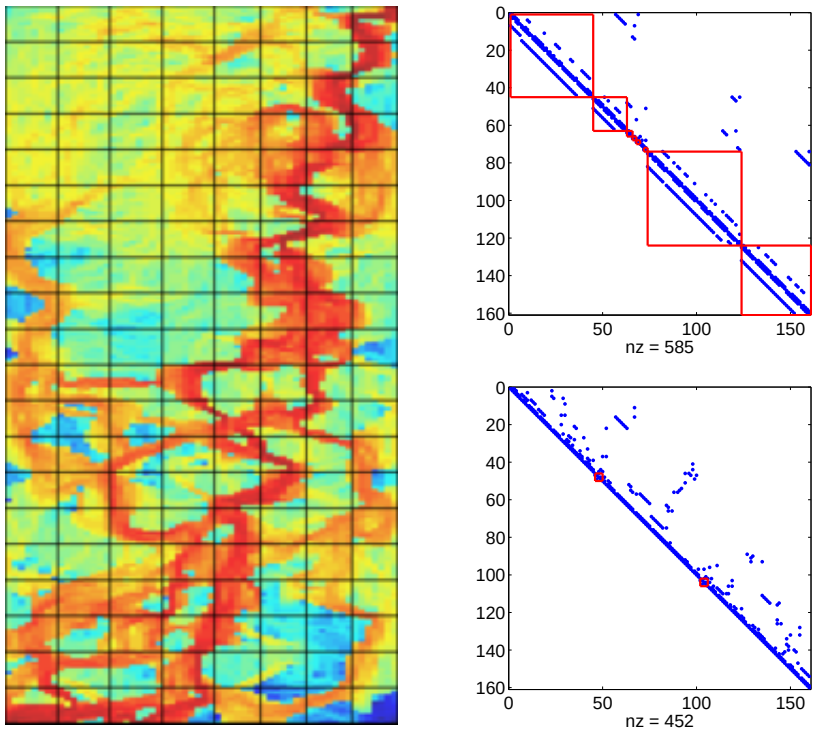
Layer 37 from SPE10

Coarse-grid discretisation: matrix structure

Flow-adapted grid

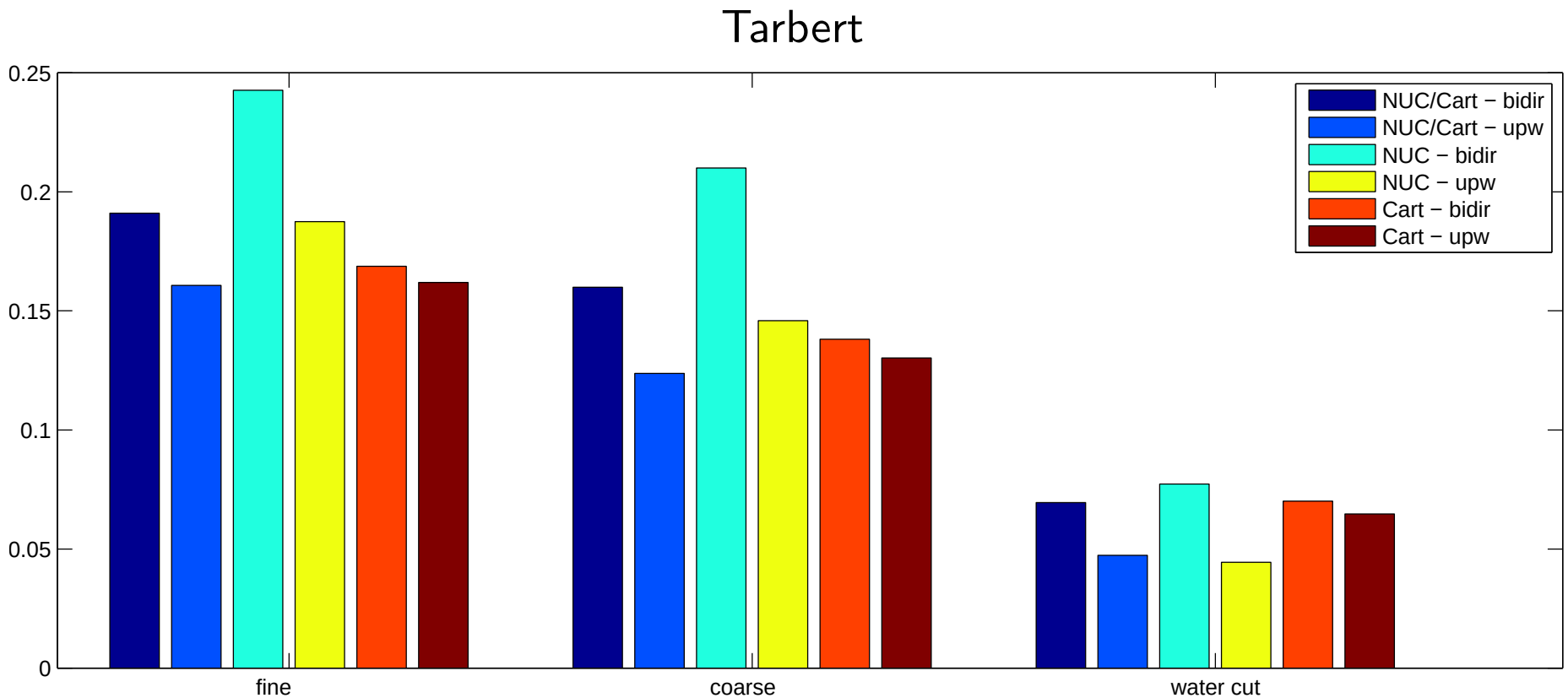


Cartesian grid



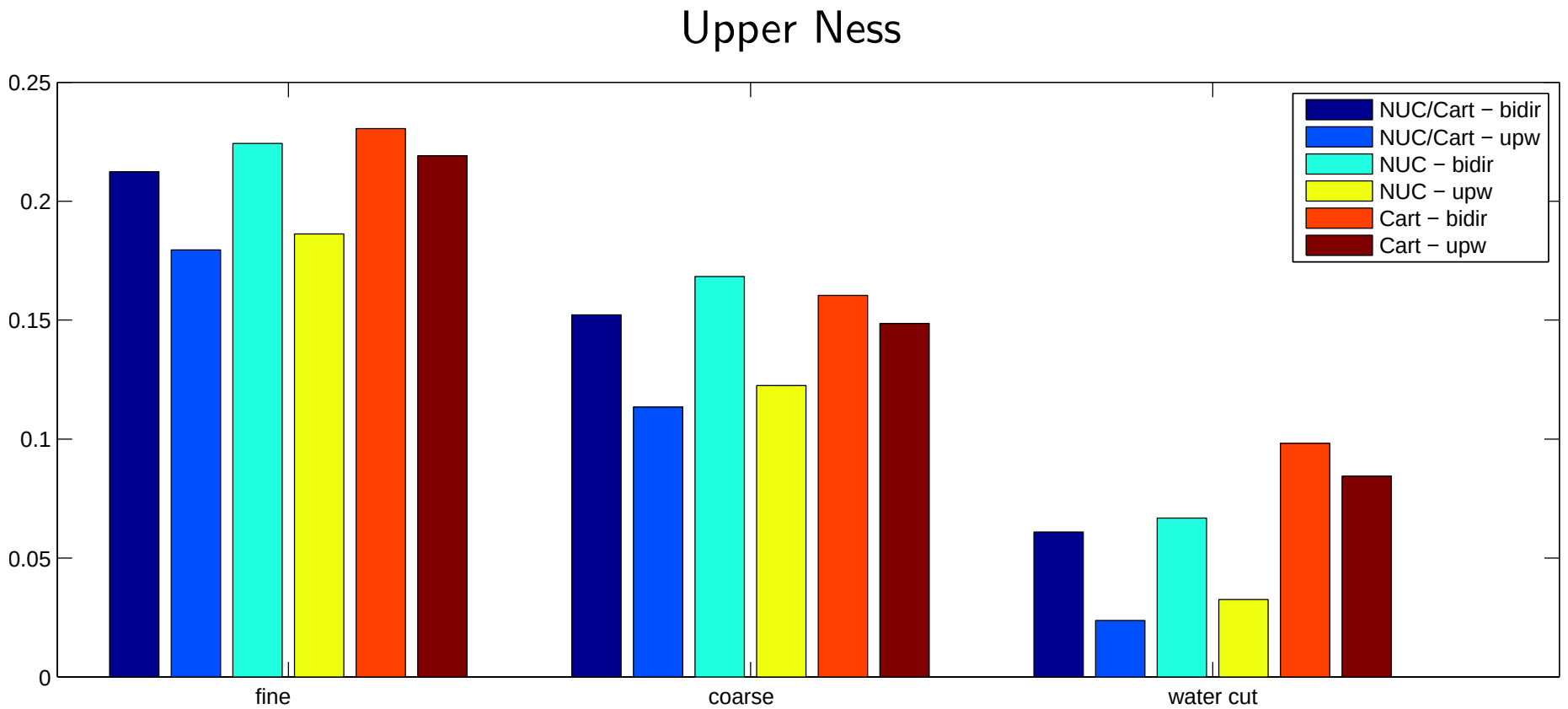
Layer 68 from SPE10. Top: bi-directional fluxes. Bottom: net fluxes

Coarse-grid discretisation: numerical errors



Average errors over all layers of the two formations in SPE10

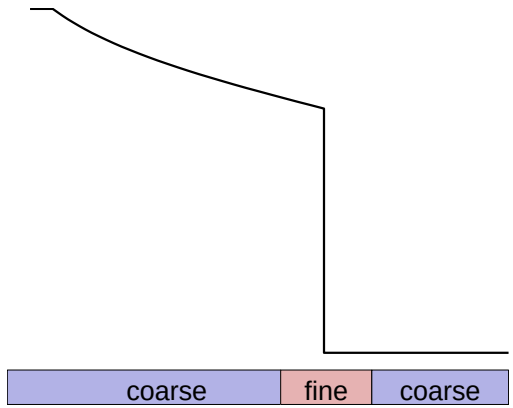
Coarse-grid discretisation: numerical errors



Average errors over all layers of the two formations in SPE10

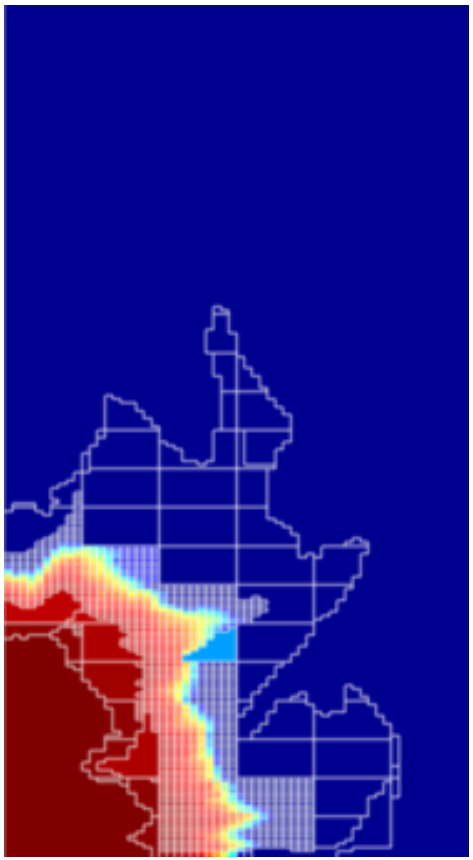
Dynamical adaption

- Inaccurate representation of strong displacement fronts can lead to significant errors.
- **Idea:** Refine dynamically around strong fronts.
- For a Buckley–Leverett displacement:
  - Unswept region ahead of the displacement: coarse grid.
  - Swept region behind the front: coarse grid.
  - At the front: fine or intermediate grid.

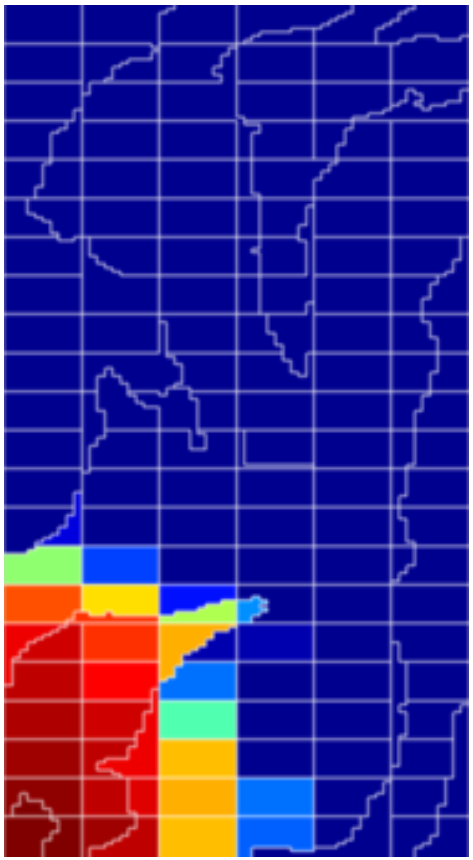


Example: layers from SPE10

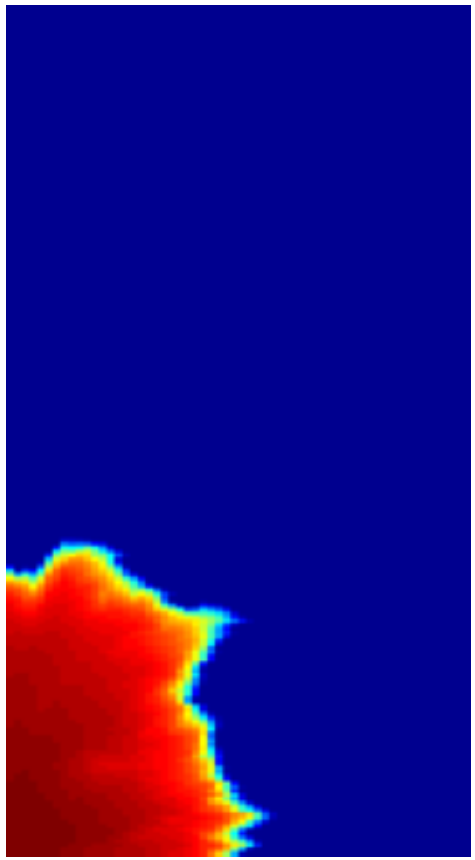
After injection of 0.1 PVI



adaptive



coarse

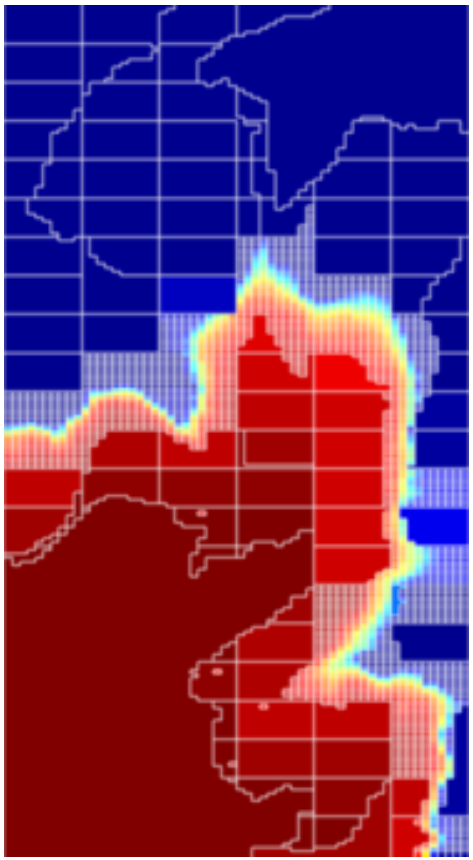


fine grid

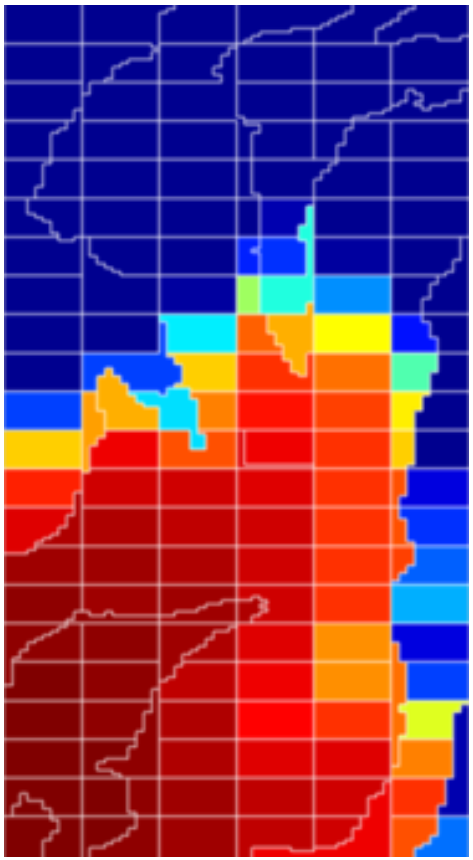
Layer 22 from SPE10

Example: layers from SPE10

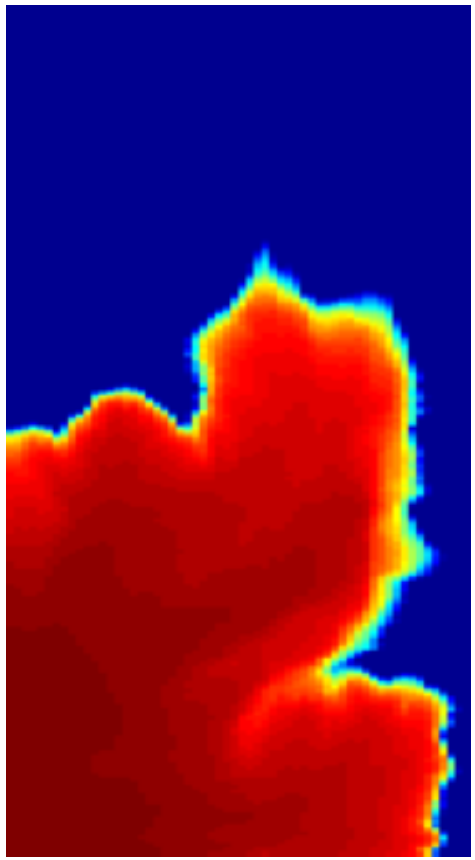
After injection of 0.5 PVI



adaptive



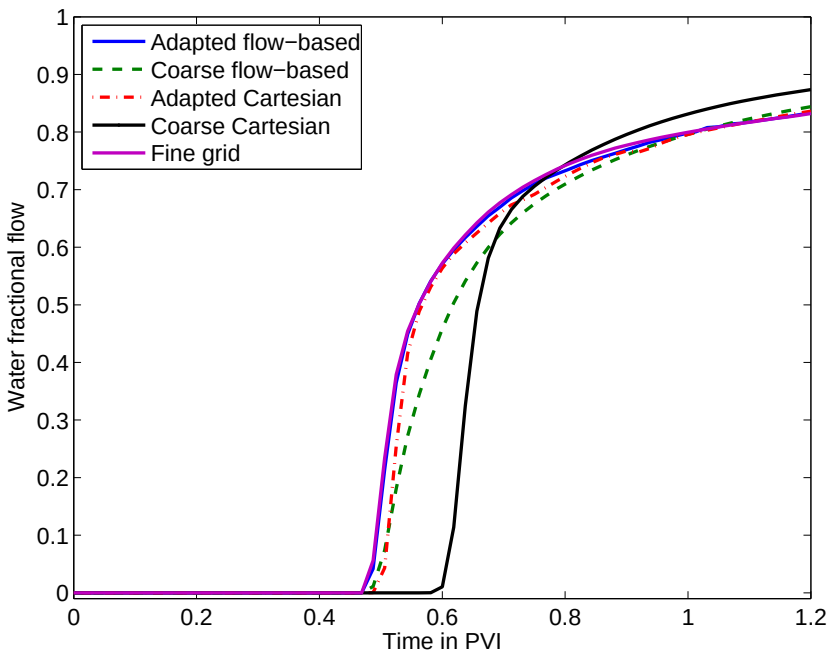
coarse

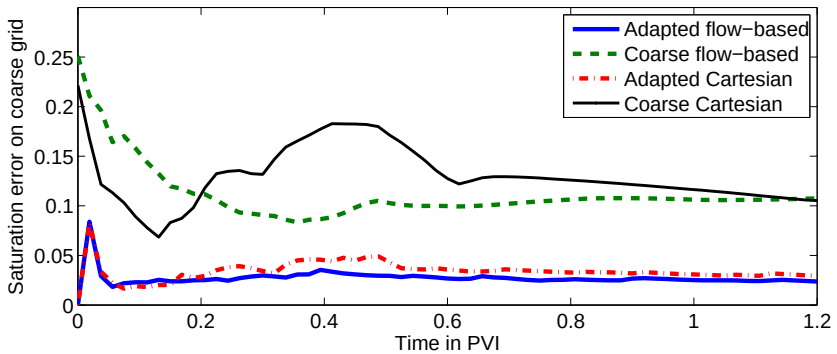


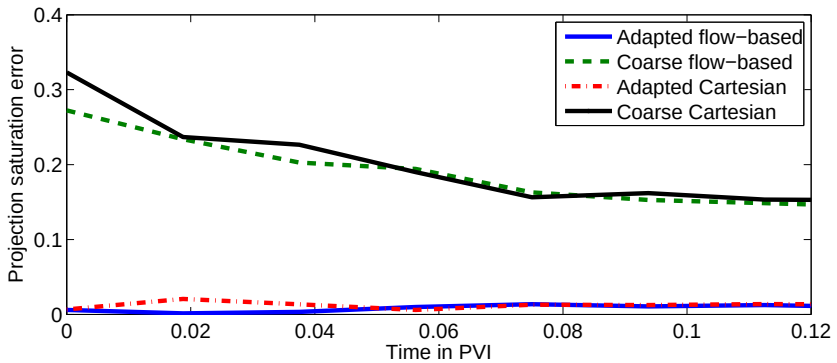
fine grid

Layer 22 from SPE10

Example: layers from SPE10







Layer 37 from SPE10