Applicability of Time Domain and Z-Domain Vector Fitting to Rational Modeling From Time Domain Responses With Consideration to Circuit Solver Integration Method

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Abstract

Linear macromodels can be extracted from simulated time domain responses, for instance using the Time Domain Vector Fitting (TD-VF) and the z-domain Vector Fitting (ZD-VF) method. In this paper we show that the TD-VF yields more accurate results than ZD-VF, mainly because of its ability to handle truncated responses. This makes TD-VF more applicable to practical applications. In addition, we show that with TD-VF one may in certain situations greatly improve the accuracy of the extracted model by introducing the same numerical integration method in the extraction routine as the one used by the circuit simulator**.**

Introduction

Electromagnetic transient simulation requires to take into account the frequency dependent effects of components. In particular, approximations based on rational functions can be used when we deal with linear components. The approach of rational macromodeling is an important tool for the extraction of compact model for use in analysis and design in both high-speed electronics system and power system. The modeling starts from a set of responses that fully characterize the terminal behavior of the device or system, given in the frequency domain or in the time domain. For that purpose, the Vector Fitting Algorithm, in its original formulation in the frequency domain (FD-VF) [1], has proved to be a robust and efficient tool.

In some cases the model is to be extracted based on time domain responses that are obtained by a circuit simulator. In recent years, two different techniques has been introduced in order to estimate the rational function starting from time domain responses: Time Domain Vector Fitting (TD-VF) [2] and Z-domain Vector Fitting [3]. The first one is a direct implementation of the VF algorithm in the time domain using convolution, whereas the second approach is nothing else than the FD-VF applied in the z-domain. In this latter case we only need to convert our data from the time domain to the z-domain.

In this work we compare the applicability of ZD-VF and TD-VF in rational approximation of time domain responses obtained by a circuit solver. After a brief description of these methods, we apply them to a high-order model, and we compare the accuracy of the extracted models in dependence on the time window length. Finally, we demonstrate that by adopting an integration scheme in TD-VF that corresponds to the one used by the circuit simulator, the accuracy of the identified model may in some situations be greatly improved.

Preliminaries on Rational Model

We consider the time domain impulse response $h(t)$ of a linear time invariant (LTI) system. For simplicity we focus on a single port structure. A generalization to multi-port cases is straightforward and will not be shown here. Let $u(t)$ and $y(t)$ denote the excitation and output response respectively. Their relation in the time domain is given by convolution (1) and in the frequency domain by product (2).

$$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)u(\tau)\,d\tau \qquad (1)$$

$$Y(s) = H(s)U(s) \qquad (2)$$

The following state space rational form is assumed for the transfer function $H(s)$, with s being the Laplace variable ($s=\sigma+j\omega$)

$$H(s) = \sum_{n=1}^{N} \frac{r_{n\_s}}{s - p_{n\_s}} + r_{0\_s} \qquad (3)$$

where $\{p_{n\_s}\}$ and $\{r_{n\_s}\}$ are the poles and residues to be identified and N indicates the model order.

Z-domain transfer function

With the introduction of the definition of the z-transform (4)

$$H(z) = \sum_{n=0}^{\infty} h[n]z^{-n} \qquad (4)$$

being $h[n]$ a discrete sequence of the time domain impulse response, we can obtain an expression for the z-domain transfer function $H(z)$ having the same structure as (3) with the only difference that the poles $\{p_{n\_z}\}$ and residues $\{r_{n\_z}\}$ are in the z-domain. The definition (4) implies an exact relation between the s-variable and the z-variable as follows

$$z = e^{s\Delta t} \qquad (5)$$

where Δt is the sampling time.

An expression of the z-transfer rational function $H(z)$ can also be obtained from the s-domain transfer function $H(s)$ by using the frequently applied bilinear transformation (6).

$$s \approx \frac{2}{\Delta t}\frac{z-1}{z+1} \qquad (6)$$

Preliminaries on Vector Fitting (VF) Algorithm

The Vector Fitting Algorithm (VF) was first introduced in [1] in the frequency domain (FD-VF). A z-domain

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counterpart version (ZD-VF) was introduced in [3]. Since both schemes follow the same construction we can summarize them by introducing a general complex variable ψ that could assume s or z values. Thus, we define a rational weighting function $\sigma(\psi)$ (7) with initial poles q_{n_ψ} and then we apply σ to the response data $\hat{H}(\psi)$ and introduce the Vector Fitting condition [1] (8).

$$\sigma(\psi) = \left(1 + \sum_{n=1}^{N} \frac{k_n}{\psi - q_{n\_\psi}}\right) \tag{7}$$

$$\sigma(\psi)\hat{H}(\psi) = m_\infty + \sum_{n=1}^{N} \frac{m_n}{\psi - q_{n\_\psi}} \tag{8}$$

Equation (8) is solved in the least squares sense and the poles of the rational function $H(\psi)$ are obtained as the zeros of the $\sigma(\psi)$ [1]. This procedure is repeated iteratively, using the new poles as starting poles until convergence. The initial poles are with FD-VF placed along the imaginary axis as complex conjugate with a small negative real part [1], and with ZD-VF inside the stable z-domain unit circle [3]. The residues of $H(\psi)$ are then obtained by setting in (8) a unity value for $\sigma(\psi)$ with the poles q_{n_ψ} equal to those obtained at the previous step.

The ZD-VF may be applied to either data in the frequency domain or in the time domain. In the first case [3], we convert the data from s-domain to z-domain with the inverse of the bilinear transformation (6). In the latter case, which is more related with the purpose of this work, a procedure is discussed in the next section.

Z-Domain VF (ZD-VF) from Time Domain Responses.

The application of the ZD-VF to time domain data requires a preliminary pre-processing of the data. The first step is the computation of the impulse transfer function $\hat{h}(t)$ that can be obtained approximately by using the excitation (9) in the circuit solver simulator. This excitation approaches the 'dirac delta' ideal function as much as finer is the time step Δt.

$$\delta^*(t) = \begin{cases} 0 & t \neq 0 \\ 1/\Delta t & t = 0 \end{cases} \tag{9}$$

Afterwards, the z-transform of $\hat{h}(t)$, called $\hat{H}(z)$, can be computed in an efficient way using the Chirp z-Transform (CZT) algorithm [8]. Next, the ZD-VF is applied to obtained poles and zeros of the rational approximation of $H(z)$. Finally, the rational model is converted from the z-domain to the s-domain by applying the bilinear approximation (6).

Time Domain VF (TD-VF)

We assume to know the excitation $u(t)$ and the output response $y(t)$ in the time domain and we want to carry out the identification of the rational $H(s)$ model (3) using the Vector Fitting Algorithm.

We start from the frequency domain formulation of the Vector Fitting. The relation (10) is introduced in (8), which relates the transfer function $\hat{H}(\psi)$ to the excitation and the output response.

$$\hat{H}(\psi) = \left.\frac{Y(\psi)}{U(\psi)}\right|_{\psi=s} \tag{10}$$

Applying the inverse Laplace transform to the modified (8) gives the Time Domain Vector Fitting formulation (TD-VF) (11) [2]

$$y(t) = \sum_{n=1}^{N} m_n \tilde{u}_n(t) - \sum_{n=1}^{N} k_n \tilde{y}_n(t) + m_\infty u(t) \tag{11}$$

where $\{m_n\}$, $\{k_n\}$ and m_∞ are our unknowns. The coefficients $\tilde{u}_n(t)$ and $\tilde{y}_n(t)$ are related to the numerical method employed in the integral discretization. This significance of the numerical method is analyzed in more detail in the next section.

After having characterized (11) for every time sample $t_k = k\Delta t$ it is possible to solve the overall overdetermined linear system in least squares sense. The computed unknowns k_n allow us to calculate the zeros of $\sigma(s)$. We obtain the final poles of $H(s)$ by iterating until convergence, similarly as in the appropriate Vector Fitting section.

Finally, the residues are computed by solving another overdetermined linear system (12) in least squares sense. This follows directly from the discretization of (1) where $h(t)$ is the time domain counterpart expression of (3).

$$y(t) = \sum_{n=1}^{N} r_{n\_s}\, \tilde{u}_n(t) + r_{0\_s}\, u(t) \tag{12}$$

Numerical Integration method

The TD-VF form (11) is obtained as a consequence of applying a particular time domain discretization to the continuous general relation (8). With several discretization schemes, it is possible to give the expression for the terms $\tilde{u}_n(t)$ and $\tilde{y}_n(t)$ as shown in (13) (we show only $\tilde{u}_n(t)$ since $\tilde{y}_n(t)$ follows the same form).

$$\tilde{u}_n(t_k) = \alpha_n \tilde{u}_n(t_{k-1}) + \lambda_n u(t_k) + \mu_n u(t_{k-1}) \tag{13}$$

The coefficients α_n, λ_n and μ_n are strictly dependent on the chosen integration method used for the discretization and on the current pole q_{n_s}.

In the original TD-VF formulation [2], the recursive convolution [4] method was adopted. In our application we have considered circuit solvers that implement two specific integration schemes: trapezoidal and backward Euler. For instance, the commercial circuit solver PSCAD/EMTDC used power systems modeling is based on the trapezoidal method, whereas the circuit simulator EMTP-RV [5] can make use of both of these. For the trapezoidal approach the coefficients of (13) are given (14).

$$\alpha_n = \frac{1 + q_{n\_s}\dfrac{\Delta t}{2}}{1 - q_{n\_s}\dfrac{\Delta t}{2}} \qquad \lambda_n = \mu_n = \frac{\dfrac{\Delta t}{2}}{1 - q_{n\_s}\dfrac{\Delta t}{2}} \tag{14}$$

Both PSCAD and EMTP solvers are based on Dommel's nodal admittance method [6]. In order to avoid unnecessary discrepancy between VF-routine and circuit solver we find that the best approach is to use the same integration method in TD-VF as that used by the circuit solver. We will discuss some consequences of this later.

Application example

We consider an open-ended high voltage cable, as shown in Fig. 1. The series resistance and inductance represent as a first approximation of a power transformer. The objective is to extract a rational model of the cable and the transformer.

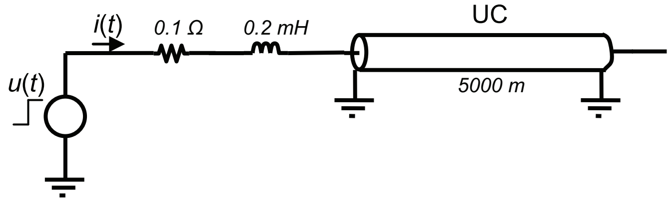

Fig. 1. Step voltage excitation of high voltage cable.

Using the circuit solver PSCAD (trapezoidal integration), we apply either a unit step or a delta voltage excitation and we calculate the $i(t)$ for a time duration of 1 ms with a time step of 1 μs. Then we apply respectively the TD-VF algorithm (with assumption of trapezoidal integration) to the unit step response, and the ZD-VF (with bilinear conversion between the z- and s-domain) to the impulse response. Both of these methods are applied with order N=30 and 30 VF iterations.

Fig. 1 compares the simulated unit step response $i(t)$ of the original circuit with the simulation result obtained by either of the two models. Clearly, a very accurate result is achieved using the model extracted by TD-VF, whereas the model obtained by ZD-VF gives substantial deviations. This is essentially due to the truncation of the time domain response used for calculating the z-domain response.

If we, however, consider a larger time window (10 ms), the current $i(t)$ is sufficiently died out to zero. Applying again the ZD-VF now gives a much better result, see Fig. 3.

The problem of the truncation effect on the computation of the z-transform is explained in Fig. 4 which shows the z-transform of the transfer function, both for a non-truncated and a truncated signal. In the latter case, the truncation results in oscillations in s-domain response which leads to a degradation in the successive fitting stage.

Fig. 5 shows how the error of the simulated response using the ZD-VF model decreases when the window time length is increased. Since the bilinear transformation is used to convert the result of the z-domain identification into the s-domain, the ZD-VF remains unable to reproduce a model with an accuracy comparable with the TD-VF (lower trace in Fig. 5).

Finally, Fig. 6 compares the frequency response of the models in the s-domain. It is seen that with ZD-VF, only the model obtained from a long time window length (10 ms) is able to match the frequency behavior of the model extracted by TD-VF.

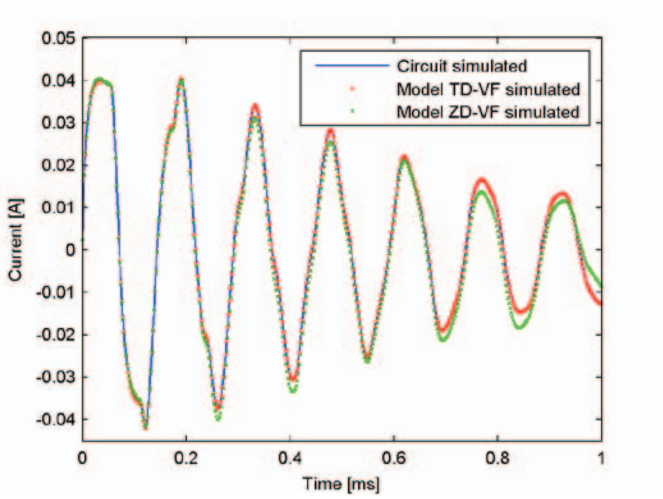

Fig. 2. Comparison between the original circuit solver simulation and simulated results by model obtained using TD-VF or ZD-VF, both with usage order 30 and a window time length of 1 ms.

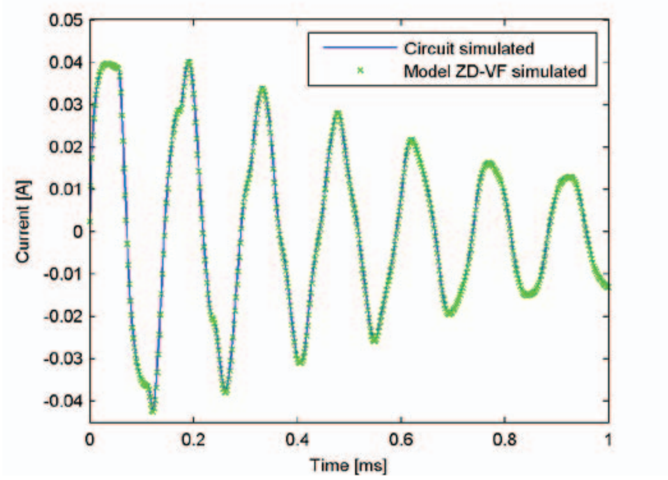

Fig. 3. Comparison between the original circuit solver simulation and simulated model result created by using ZD-VF with order 30 and a window time length of 10 ms.

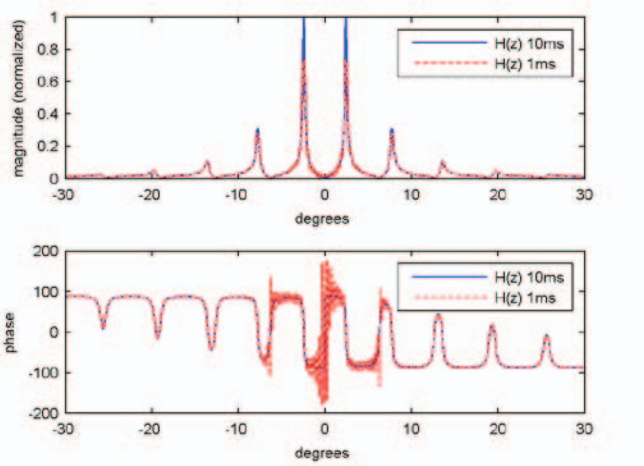

Fig. 4. Comparison between the transfer function in the z-domain $H(z)$ of a non-truncated response (10 ms) and a truncated response (1 ms).

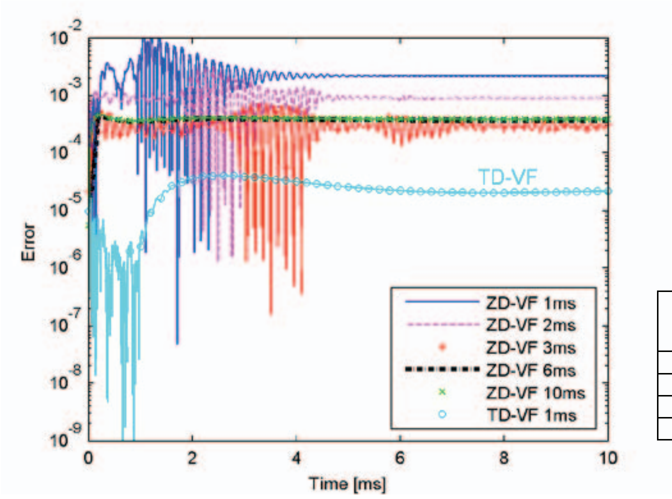

Fig. 5. Deviation (absolute error) between the original circuit solver simulation and the simulated models obtained using different window time lengths.

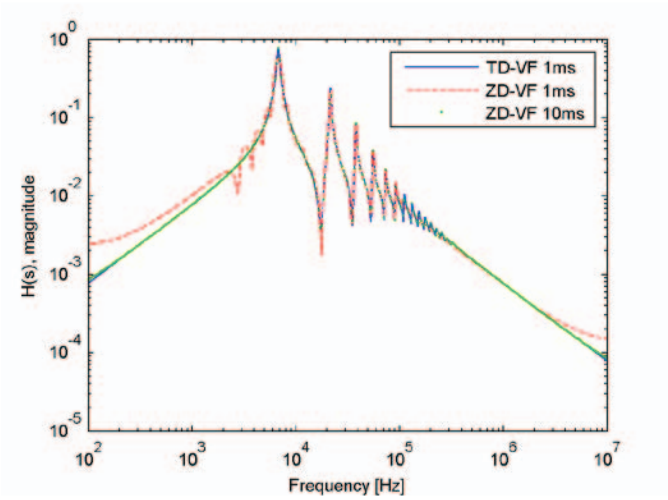

Fig. 6. Comparison of the models brought in the s-domain for TD-VF and ZD-VF (2 different window time lengths).

Further Discussion on Integration Method

We consider the driving point admittance of a series RLC circuit with R=1 Ω, L=1 mH, and C=1 μF. The admittance transfer function has an analytical complex pole pair of (–500±j31619).

Using the circuit solver EMTP-RV, we compute the transient current due to a step voltage excitation with usage either the trapezoidal or the backward Euler method. TD-VF is next applied to the simulated current in order to recover the poles of the admittance transfer function using alternative integration methods. The results in Table I and Table II show that the accuracy of the extracted model can be greatly improved by using the same integration method in TD-VF as that used by the circuit simulator for generating the time domain responses.

The results obtained here are quite general, even when considering more complex structures, but with some limitations. Firstly, a well-defined integration method must be used by the circuit simulator, and it is in any case not possible to include cases which include travelling wave models or non-linear devices. Also, the exact identification requires that the poles and residues are observable in the given time domain responses and that the fitting order is sufficiently high. Otherwise, we merely obtain an approximation of the analytical poles and residues.

TABLE I
POLES OBTAINED BY TD-VF. APPLICATION TO SIMULATION RESULT OBTAINED BY TRAPEZOIDAL-BASED CIRCUIT SOLVER

Iter. count	Trapezoidal	Backward Euler	Recursive Conv.
0	-1.9e3±j1.9e5	-1.9e3±j1.9e5	-1.9e3±j1.9e5
1	-500±j31619	3.9e3±j499.9	0.0058
3	-500±j31619	3.9e3±j499.9	298±j6738
5	-500±j31619	3.9e3±j499.9	1414±j6924

TABLE II
POLES OBTAINED BY TD-VF. APPLICATION TO SIMULATION RESULT OBTAINED BY BACKWARD EULER-BASED CIRCUIT SOLVER

Iter. count	Trapezoidal	Backward Euler	Recursive Conv.
0	-1.9e3±j1.9e5	-1.9e3±j 1.9e5	-1.9e3±j1.9e5
1	-2974±j31345	-500±j31619	-2742±j30126
3	-2974±j31345	-500±j31619	-2956±j31282
5	-2974±j31345	-500±j31619	-2956±j31282

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