Causality Preserving Passivity Enforcement for Traveling-Wave-Type Transmission-Line Models

Dirk Deschrijver, *Member, IEEE*, Bjørn Gustavsen, *Senior Member, IEEE*, and Tom Dhaene, *Senior Member, IEEE*

Abstract—Transmission-line models must be passive in order to guarantee stable time-domain simulations. Enforcing the passivity by adding a conductance matrix to the line model has been proposed, which is established by a frequency sweep. This leads to instantaneous coupling between the two line ends, thus violating the causality of the line. This letter shows that the coupling terms can be removed with only a negligible change to the model's perturbation. The removal of coupling terms makes the approach more suitable for the application with existing transmission-line models.

Index Terms—Electromagnetic transients, instability, passivity, transmission-line model, universal line model.

I. INTRODUCTION

TRANSMISSION-LINE models for transient studies are usually based on the traveling-wave method. It is well known that such models can lead to unstable time-domain simulations due to passivity violations.

In [1], enforcing passivity by connecting a conductance matrix of size $2n \times 2n$ to the line ends was proposed, where n is the number of phase conductors. The two offdiagonal blocks are nonzero which provides instantaneous coupling between the line ends, thereby violating the delay property of the line and, thus, its causality. The coupling is undesirable since it requires modifying the structure of the Norton equivalent used for interfacing the line model to the circuit simulator. In addition, some circuit simulators take advantage of the decoupling to increase the computational efficiency of simulations involving nonlinear devices.

This letter shows that the procedure in [1] is also applicable with the coupling terms set equal to zero. Some numerical results illustrate that the additional deviation caused by the enforcement is quite negligible when compared to a perturbation of the full nodal admittance matrix.

II. PASSIVITY CRITERION

The nodal admittance matrix is a block symmetric matrix

$$Y_n = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \tag{1}$$

where A and B are computed directly from the rational approximations of Y_c and H [1]. These matrices are obtained from the series impedance and the shunt admittance of the line

$$A = Y_c(I + H^2)(I - H^2)^{-1}$$
$$B = -2Y_c H(I - H^2)^{-1}. \tag{2}$$

Due to the fitting errors associated with Y_c and H, it follows that Y_n has a slightly unsymmetrical structure. In order to avoid unstable time-domain simulations, the following passivity condition must be satisfied for all frequencies $s = j\omega$:

$$\text{eig}(Y_H) = \text{eig}\left(\left(Y_n(s) + Y_n^H(s)\right)/2\right) \geq 0, \quad \forall s. \tag{3}$$

III. PASSIVITY COMPENSATION

To enforce the passivity of the model, the matrix Y_H is swept over a given band of frequency samples, and an updated correction term $\Delta Y_{n,\text{corr}}$ is computed for each frequency [1].

A pseudocode overview of the algorithm is shown in Fig. 1.

At each frequency s_k (for $k = 1, \ldots, N_s$), the matrix $Y_H(s_k)$ is factored into two modal decompositions that contain the positive and the negative eigenvalues (line 1). To satisfy the passivity condition (3), only the eigenvalues of $Y_H(s_k)$, which are positive or zero, should be retained. Therefore, the real part of the negated second term can be taken as a correction term (line 2). A small modification is introduced to enforce the symmetry of the correction term explicitly (lines 3 and 4). This correction step at a given frequency may have to be repeated a few times, because the imaginary part of the correction term is discarded to ensure that $\Delta Y_{n,\text{corr}}$ is a real symmetric matrix.

This process is applied for each frequency, while taking the previously calculated corrections into account (line 5).

IV. COMPENSATING ONLY THE DIAGONAL BLOCKS

In order to avoid the causality violations, only the diagonal blocks A of Y_n should be affected by the correction. This is enforced by making a modification to line 4 of the algorithm, such that the offdiagonal blocks $\Delta B_{\text{corr}}(s_k)$ are discarded.

V. EXAMPLE: THREE SINGLE CORE COAXIAL CABLES

The presented method is applied to model three 145-kV single-core coaxial cables. The properties and configuration of the cable system are reported in [1, Sec. VI]. An accurate rational approximation of Y_c and H is calculated and the nodal admittance matrix Y_n is formed (2). The eigenvalue curves of Y_H are then visualized by Fig. 3 over the frequency range (0.01 Hz–10 kHz). It is seen that some of the eigenvalues are negative, leading to several small passivity violations.

The algorithm in Fig. 1 is applied to compute a correction term $\Delta Y_{n,\text{corr}}$ that modifies the A and B blocks of Y_n. This enforces the eigenvalues to become positive, as shown by the

$$\Delta Y_{n,corr} := \begin{pmatrix} \Delta A_{corr} & \Delta B_{corr} \\ \Delta B_{corr} & \Delta A_{corr} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

for $k=1:Ns$

1. $\quad Y_H(s_k) + \Delta Y_{n,corr} \rightarrow T\Lambda_{pos}T^{-1} + T\Lambda_{neg}T^{-1}$

2. $\quad \Re e\left(-T\Lambda_{neg}T^{-1}\right) \approx \begin{pmatrix} \Delta A(s_k) & \Delta B(s_k) \\ \Delta B(s_k) & \Delta A(s_k) \end{pmatrix}$

3. $\quad \Delta A_{corr}(s_k) := (\Delta A(s_k) + \Delta A(s_k)^{\mathrm{T}})/2$

4. $\quad \Delta B_{corr}(s_k) := (\Delta B(s_k) + \Delta B(s_k)^{\mathrm{T}})/2$

5. $\quad \Delta Y_{n,corr} := \Delta Y_{n,corr} + \begin{pmatrix} \Delta A_{corr}(s_k) & \Delta B_{corr}(s_k) \\ \Delta B_{corr}(s_k) & \Delta A_{corr}(s_k) \end{pmatrix}$

end

Fig. 1. Pseudocode for calculating the correction term $\Delta Y_{n,\mathrm{corr}}$.

4. $\quad \Delta B_{corr}(s_k) = 0$

Fig. 2. Modification to the pseudocode algorithm in Fig. 1.

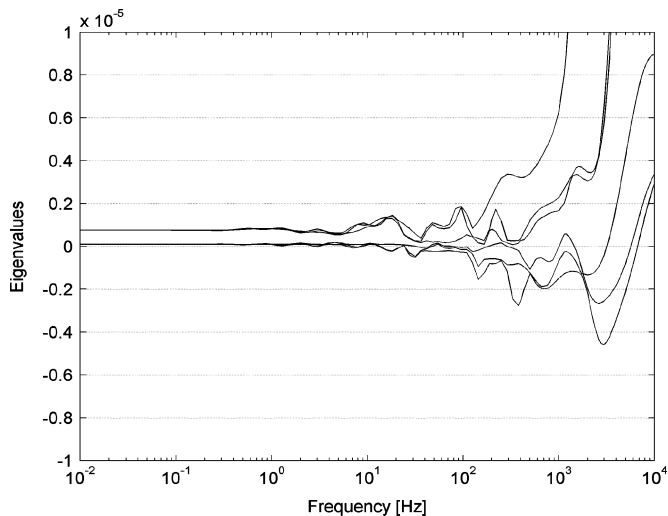Fig. 3. Eigenvalues of Y_H before passivity correction.

Fig. 4. Eigenvalues of Y_H after passivity correction.

blue solid line in Fig. 4. If only the diagonal blocks A of Y_n are modified (new procedure), then the deviation to the eigenvalue curves is quite comparable, as shown by the red dotted line in Fig. 4. This confirms that the removal of the coupling terms introduces a change that is negligibly small.

VI. DISCUSSION

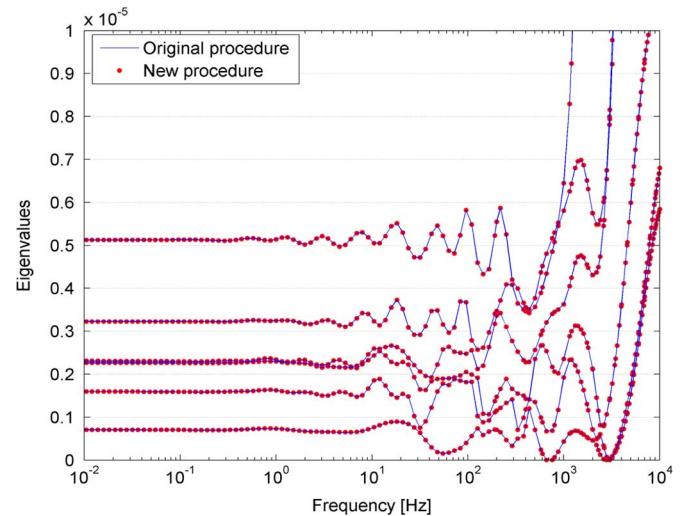The result in Fig. 4 warrants an explanation. The original procedure is found to approximately give a correction Δ to A and B, while the new procedure is found to give a correction 2Δ to A. The (violating) small eigenvalues occur at low frequencies where they correspond to voltage applications that are equal at the two line ends, thus corresponding to charging currents. It follows from the relation $i = Y_n \cdot v$ that with such a voltage application, the perturbation to the current response will be equal by the two approaches. Since only A is perturbed by the new approach, the eigenvalues associated with short-circuit conditions will be higher than with the original approach, but this does not matter since the losses with such voltage applications are very high.

The actual implementation of the correction factor in a transmission-line model is straightforward since the conductance matrix A becomes added to the two conductance matrices of the Norton equivalent. In the case of a phase-domain model, the perturbation can be tapered off at low and high frequencies with a band-pass filter that introduces two new pole-residue terms [1] in the rational approximation for Y_c.

VII. CONCLUSION

A robust algorithm for passivity enforcement of traveling-wave-type transmission-line models is proposed. The algorithm modifies the diagonal block elements of the nodal admittance matrix to ensure causal, stable time-domain simulations. The additional deviation introduced by the enforcement is negligible when compared to a perturbation of the full nodal admittance matrix.

REFERENCES

[1] B. Gustavsen, "Passivity enforcement for transmission line models based on the method of characteristics," *IEEE Trans. Power Del.*, vol. 23, no. 4, pp. 2286–2293, Oct. 2008.