Power Engineering Letters

On Passivity Tests for Unsymmetrical Models

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***Abstract*—Rational models (not including explicit sources as in the case of generators) must be passive in order to guarantee stable simulations. Passivity tests for use with perturbation-type passivity enforcement methods are usually based on frequency sweeping or calculation of the eigenvalues of a Hamiltonian matrix derived from the model's parameters. Recently, half-size test matrices have been derived which reduce the computation time of eigenvalues by a factor of about eight. In this paper, we elaborate on the passivity assessment for unsymmetrical models. We show that the half-size test matrix approach is valid only for the symmetrical case, while the full-size Hamiltonian must be used for unsymmetrical models.**

***Index Terms*—Passivity assessment, passivity enforcement, pole-residue model, rational model, state-space model.**

I. Introduction

RATIONAL modeling is a convenient way of representing linear components with frequency-dependent behavior. The modeling amounts to approximating ("fitting") a set of characterizing responses by rational functions, leading to a state-space model. In order to ensure stable time domain simulations, the model must be passive. The passivity property can be enforced by subjecting the extracted model to passivity enforcement by perturbation, based on assessment of the model's passivity characteristics.

Most of the literature on passivity assessment is focused on symmetrical models. This is justified by the fact that physical systems result in symmetrical port matrices. The symmetry is retained in the modeling by fitting a pole-residue model to the data. However, columnwise fitting is sometimes used to reduce computation time. This leads to an unsymmetrical model.

In this letter we investigate passivity assessment tests for unsymmetrical admittance-based models, based on frequency sweeping and on test matrices. We also rectify an error in [1] regarding the use of the half-size test matrices.

II. Admittance-Based Modeling

We focus on models that represent the physical device or system by its admittance matrix $\mathbf{Y}$, which relates port voltages $\mathbf{v}$ to port currents $\mathbf{i}$. $\mathbf{Y}$ is a symmetrical, complex-valued matrix $\mathbf{Y}=\mathbf{Y}^T$

$$\mathbf{i}(j\omega)=\mathbf{Y}(j\omega)\mathbf{v}(j\omega). \quad (1)$$

A symmetrical pole-residue model (2) can be obtained from a set of discrete data $(s,\mathbf{Y}(s))$ by subjecting the upper (or lower) triangle of $\mathbf{Y}$ to vector fitting [3] with a common pole set. The model (2) can be expanded into a state-space model (3).

One may also subject the columns of $\mathbf{Y}$ to rational fitting using a private pole set for each column. This gives a state-space model (3) whose admittance matrix is unsymmetrical

$$\mathbf{Y}(j\omega)\cong\sum_{m=1}^{N}\frac{\mathbf{R}_m}{j\omega-a_m}+\mathbf{D} \quad (2)$$

$$\mathbf{Y}(j\omega)\cong\mathbf{C}(j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}. \quad (3)$$

III. Passivity Tests for Unsymmetrical Models

A. Frequency Sweeping

The (active and reactive) power consumed by the model is given by (4), where superscript H denotes the Hermitian (transpose and conjugate). The conjugate of (4) is given by (5). Adding (4) and (5) leads to the active power losses (6)

$$p+jq=\mathbf{i}^H\mathbf{v}=\mathbf{v}^H\mathbf{Y}^H\mathbf{v} \quad (4)$$

$$p-jq=\mathbf{v}^H\mathbf{i}=\mathbf{v}^H\mathbf{Y}\mathbf{v} \quad (5)$$

$$p=\frac{1}{2}\mathbf{v}^H(\mathbf{Y}+\mathbf{Y}^H)\mathbf{v}. \quad (6)$$

This shows that the passivity test could be obtained by the matrix $\mathbf{Y}_H$ (7) with the requirement that all of its eigenvalues be positive (8). $\mathbf{Y}_H$ has all of its eigenvalues real since it is a Hermitian matrix. In the case of symmetrical models, the symmetry of $\mathbf{Y}$ results in $\mathbf{Y}_H$ becoming equal to $\mathbf{G}=\mathrm{Re}\{\mathbf{Y}\}$

$$\mathbf{Y}_H(j\omega)=\frac{1}{2}\left(\mathbf{Y}(j\omega)+\mathbf{Y}^H(j\omega)\right)=\frac{1}{2}\left(\mathbf{Y}(j\omega)+\mathbf{Y}^T(-j\omega)\right) \quad (7)$$

$$\mathrm{eig}\left(\mathbf{Y}_H(j\omega)\right)>0. \quad (8)$$

B. Test Matrices

Reference [1] introduced a new test matrix $\mathbf{S}$ (9) for passivity assessment which is half the size of the Hamiltonian matrix [2] that has traditionally been used. $\mathbf{S}$ identifies via the square-root of its positive-real eigenvalues the crossover frequencies where

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eigenvalues of $\mathbf{G}$ change sign

$$\mathbf{S} = \mathbf{A}(\mathbf{B}\mathbf{D}^{-1}\mathbf{C} - \mathbf{A}). \tag{9}$$

Since its derivation assumed that $\mathbf{Y}$ is symmetrical, we now derive the test matrix for the unsymmetrical case.

The state equations associated with the model (3) are

$$j\omega \mathbf{x}_1 = \mathbf{A}\mathbf{x}_1 + \mathbf{B}\mathbf{u} \tag{10a}$$
$$\mathbf{y}_1 = \mathbf{C}\mathbf{x}_1 + \mathbf{D}\mathbf{u}. \tag{10b}$$

For the second term of $\mathbf{Y}_H$ in (7), we get the model (11) whose state space equations are given by (12)

$$\mathbf{Y}^T(-j\omega) = \mathbf{B}^T(-j\omega\mathbf{I} - \mathbf{A}^T)^{-1}\mathbf{C}^T + \mathbf{D}^T \tag{11}$$
$$-j\omega \mathbf{x}_2 = \mathbf{A}^T\mathbf{x}_2 + \mathbf{C}^T\mathbf{u} \tag{12a}$$
$$\mathbf{y}_2 = \mathbf{B}^T\mathbf{x}_2 + \mathbf{D}^T\mathbf{u}. \tag{12b}$$

For the model (7), we are interested in finding the input $\mathbf{u}$ which makes the (real) output $\mathbf{g}$ zero

$$\mathbf{g} = \frac{1}{2}(\mathbf{y}_1 + \mathbf{y}_2) = \frac{1}{2}\left(\mathbf{C}\mathbf{x}_1 + \mathbf{B}^T\mathbf{x}_2 + (\mathbf{D} + \mathbf{D}^T)\right)\mathbf{u} = 0. \tag{13}$$

This gives

$$\mathbf{u} = -(\mathbf{D} + \mathbf{D}^T)^{-1}(\mathbf{C}\mathbf{x}_1 + \mathbf{B}^T\mathbf{x}_2). \tag{14}$$

By introducing the auxiliary variable $\mathbf{E}$ (15) and inserting (14) into (10a) and (12a), we get (16) which in matrix form becomes (17)

$$\mathbf{E} = (\mathbf{D} + \mathbf{D}^T)^{-1} \tag{15}$$
$$j\omega \mathbf{x}_1 = (\mathbf{A} - \mathbf{B}\mathbf{E}\mathbf{C})\mathbf{x}_1 - \mathbf{B}\mathbf{E}\mathbf{B}^T\mathbf{x}_2 \tag{16a}$$
$$-j\omega \mathbf{x}_2 = -\mathbf{C}^T\mathbf{E}\mathbf{C}\mathbf{x}_1 + (\mathbf{A}^T - \mathbf{C}^T\mathbf{E}\mathbf{B}^T)\mathbf{x}_2 \tag{16b}$$

$$\begin{bmatrix} (\mathbf{A}-\mathbf{B}\mathbf{E}\mathbf{C})-j\omega\mathbf{I} & \mathbf{B}\mathbf{E}\mathbf{B}^T \\ -\mathbf{C}^T\mathbf{E}\mathbf{C} & (-\mathbf{A}^T+\mathbf{C}^T\mathbf{E}\mathbf{B}^T)-j\omega\mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ -\mathbf{x}_2 \end{bmatrix} = 0. \tag{17}$$

Equation (17) is recognized as an eigenvalue problem where the matrix is (18) (after replacing $\mathbf{E}$ with (15)), shown at the bottom of the page.

$\mathbf{M}$ is identical to the Hamiltonian matrix that has traditionally been used for passivity assessment [2], and crossover frequencies for the eigenvalues of $\mathbf{Y}_H$ appear as the imaginary eigenvalues of $\mathbf{M}$. Thus, the test matrix $\mathbf{S}$ developed in [1] is only applicable to symmetrical matrices.

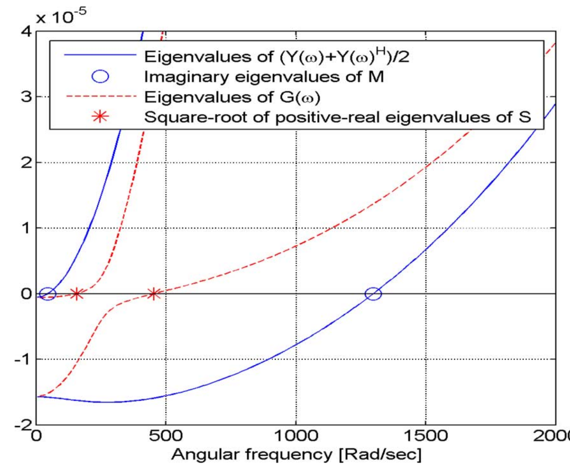

Fig. 1. Passivity assessment of unsymmetrical model.

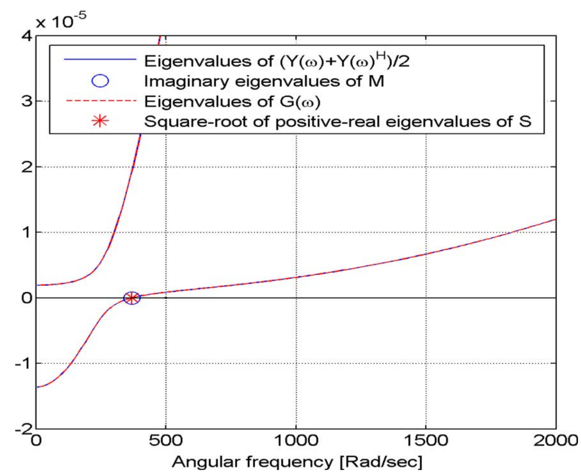

Fig. 2. Passivity assessment of symmetrical model.

IV. Example

The 2 × 2 admittance matrix of a 10 km underground cable has been calculated with respect to the core and screen conductors at the near end with the far end open. Figs. 1 and 2 show the result from passivity assessment when $\mathbf{Y}_{\text{data}}$ is subjected to rational approximation by vector fitting [3], using either columnwise fitting or pole-residue fitting.

With columnwise fitting (Fig. 1), the resulting model is slightly unsymmetrical. It is observed that the imaginary eigenvalues of $\mathbf{M}$ (18) identify the crossover frequencies of the eigenvalues of $\mathbf{Y}_H$, whereas the square-root of the positive-real eigenvalues of $\mathbf{S}$ (9) identify the crossover frequencies of the eigenvalues of $\mathbf{G} = \mathrm{Re}\{\mathbf{Y}\}$. However, the passivity assessment by $\mathbf{G}$ and $\mathbf{S}$ is incorrect.

$$\mathbf{M} = \begin{bmatrix} (\mathbf{A} - \mathbf{B}(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{C}) & \mathbf{B}(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{B}^T \\ -\mathbf{C}^T(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{C} & (-\mathbf{A}^T + \mathbf{C}^T(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{B}^T) \end{bmatrix} \tag{18}$$

With pole-residue fitting (Fig. 2), the resulting model (3) is symmetrical. The eigenvalues of $\mathbf{Y}_H$ and $\mathbf{G}$ are now identical and so their crossover frequencies are correctly identified via both $\mathbf{M}$ and $\mathbf{S}$ (but, of course, much faster with the latter).

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