Time Delay Identification for Transmission Line Modeling

Bjørn Gustavsen
SINTEF Energy Research
N-7465 Trondheim, Norway.
bjorn.gustavsen@sintef.no

Abstract

Frequency dependent transmission line models of the traveling wave type requires to approximate the propagation function by a rational function plus a time delay. It is shown that to simply use the time delay of lossless propagation can lead to significant loss of accuracy of the rational approximation. A procedure is shown which optimizes the time delay together with the poles and residues of the rational approximation. This is achieved by combining Brent's method with Vector Fitting.

Introduction

Frequency dependent transmission line models have been used in the simulation of power system transients since the 1970s [1],[2],[3] and are today also being used for the simulation of high-speed interconnects. The traveling wave type formulation (method of characteristics) is by far the most popular approach, due to its computational efficiency, accuracy, and stability. Normally, the model is identified in the frequency domain via rational function approximation, which leads to a highly efficient time domain implementation by recursive convolutions [1].

The traveling wave models are formulated in terms of a characteristic admittance Y_c and a propagation function H, which need to be approximated by rational functions. The fitting of Y_c is in general straightforward and can be achieved by a relatively low order. The fitting of H is more difficult, because one also needs to associate a time delay which affects the quality of the rational approximation.

This paper shows a procedure for calculating the time delay which for a given order of approximation gives the smallest fitting error. The procedure is based on Golden Section Search and Parabolic Interpolation as combined in Brent's Method [4]. Brent's method is used together with with Vector Fitting [5] which for a given time delay produces poles and residues which minimize the least-squares error.

Traveling Wave Model

Consider an *n*-conductor transmission line of parallel conductors. In the frequency domain, the per unit length parameters of the line are given as

$$-\frac{dV}{dx} = Z \cdot I \tag{1}$$

$$-\frac{dI}{dx} = Y \cdot V \tag{2}$$

$$Z(\omega) = R(\omega) + j\omega L(\omega) \tag{3}$$

$$Y(\omega) = G(\omega) + j\omega C(\omega) \tag{4}$$

where Z and Y are *nxn* matrices.

Combining (1) and (2) gives the traveling wave equation whose solution can be expressed in terms of forward and backward waves. With line ends denoted *k* and *m*, the solution based on current waves can be written as

$$I_k = Y_c \cdot V_k - 2H \cdot I_{mr} \tag{5}$$

$$I_m = Y_c \cdot V_m - 2H \cdot I_{kr} \tag{6}$$

where subscript *r* denotes reflected wave, see fig. 1. Y_c and H are the characteristic admittance and propagation function. They can be calculated by the expressions

$$Y_c = Z^{-1}\sqrt{Z \cdot Y} \tag{7}$$

$$H = \exp(-\sqrt{Y \cdot Z} \cdot l) \tag{8}$$

The time domain calculations are carried out using convolutions between the impulse responses $y_c(t)$ and $h(t)$, and the associated driving functions ($v(t)$, $i_r(t)$).
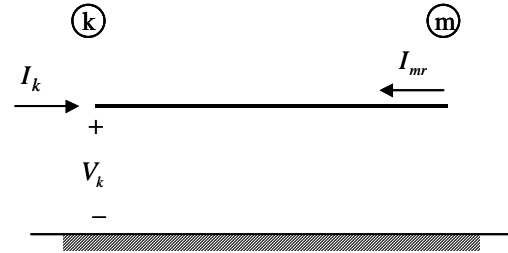

Fig. 1. Traveling waves on *n*-conductor line

Rational Approximation

In what follows we will consider only single conductor systems, which means that H and Y_c are scalar functions of frequency. The treatment of multi-conductor systems is discussed in a later section.

In order to achieve high computational efficiency of the time domain convolutions, it is necessary to approximate Y_c and and H with rational functions. Vector Fitting [5] (VF) is eminently suited for this pupose, due to its accuracy, high computational efficiency, and enforcement of guaranteed stable poles.

Application of VF to Y_c gives a rational approximation on the desired form

$$Y_c = \sum_{m=1}^{N_{Y_c}} \frac{R_m}{s - a_m} + D \tag{9}$$

The propagation function H is to be fitted with a rational function plus a time delay

$$H = \left(\sum_{m=1}^{N_H} \frac{R_m}{s - a_m}\right) \cdot e^{-j\omega\tau} \tag{10}$$

A difficulty in calculating the approximation (10) is that the time delay τ is one of the unknowns, along with poles $\{a_m\}$ and residues $\{R_m\}$. This makes direct application of VF to (10) impossible. If, however, the time delay is taken as a

known quantity, the poles and residues can be calculated by applying VF to the alternative problem

$$H \cdot e^{j\omega\tau} = \sum_{m=1}^{N_H} \frac{R_m}{s - a_m} \qquad (11)$$

Significance of Time Delay

As an example we consider a system of a 25 km single conductor over a lossy ground, see fig. 2. The series impedance Z was calculated taking into account skin effect in the conductor and the ground. Fig. 3 shows the magnitude function of H in the frequency range 1 Hz–10 MHz.

$$R_{DC} = 1.052 \cdot 10^{-4} \ \Omega/\text{m}$$
$$r = 11 \cdot 10^{-3} \ \text{m}$$

11 m

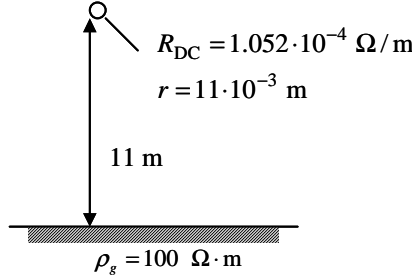$$\rho_g = 100 \ \Omega \cdot \text{m}$$
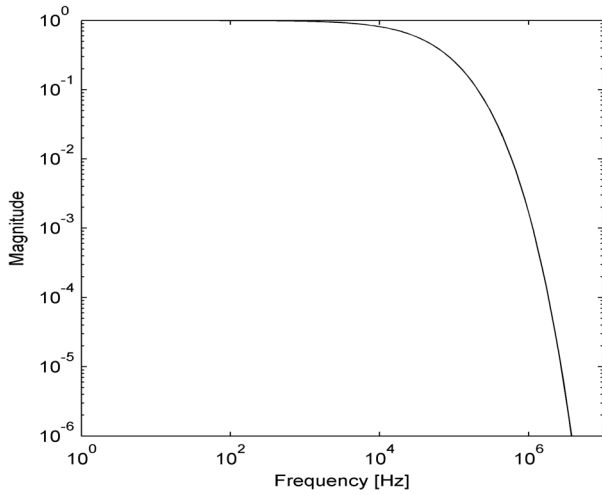
Fig. 2. Lossy conductor over lossy ground

Fig. 3. Magnitude function of H

H was next multiplied with the factor $\exp(j\omega\tau)$ as shown in (11) and subjected to rational fitting by VF as implemented in the public domain routine `vectfit.m`. Fig. 4 shows the rms-error of (11) as function of the time delay, with N=10. The rms-error is shown when only stable poles are used (kill=2 in `vectfit.m`), and when unstable poles are allowed (kill=0). For reference, the lossless time delay of $25E3/3E8$=83.33 μs is shown in the plot by the vertical line. It is seen that the minimum rms-error occurs at a time delay which is larger than the lossless time delay. As the time delay is increased significantly beyond the lossless time delay, the approximation with only stable poles gives a substantially higher rms-error than the one allowing unstable poles. This result occurs because usage of a too large time delay results in a function having a phase angle *smaller* (more positive) than that of the minimum-phase shift function of the given magnitude function, thus requiring unstable poles to be fitted with decent accuracy.
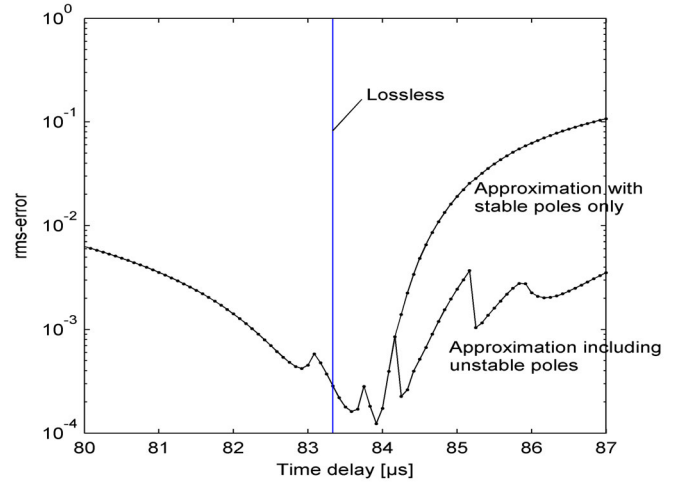

Fig. 4. Effect of time delay on rms-error of rational approximation (N=10).

Fig. 5 shows the rms-error of (11) when allowing stable poles only, with different orders of approximation: N=5, 10 and 15. It can be seen that the time delay giving the minimum rms-error decreases as the order of approximation is increased.

It can thus be concluded that the time delay which produces the smallest rms-error of the approximation is in general larger than the lossless time delay. Thus, τ should be subject to optimization.
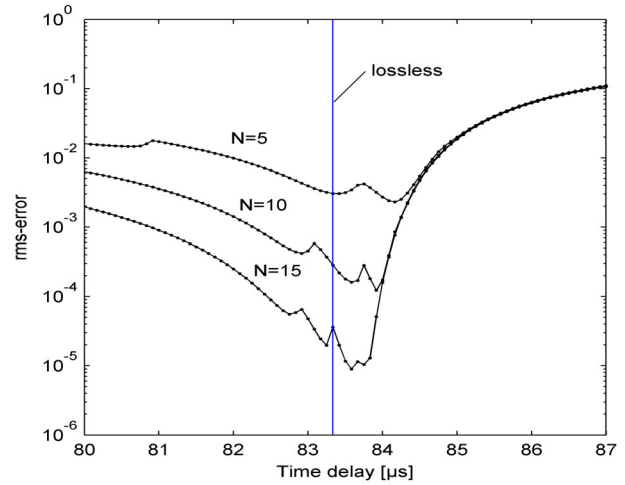

Fig. 5. Effect of order (N) on rms-error. Stable poles only.

Optimization of Time Delay

A. Brent's Method

A practical approach is to systematically search for the time delay which gives the smallest possible rms-error of (11). This can be done by subjecting $H \cdot \exp(j\omega\tau)$ to VF, which permits to calculate the rms-error as function of the time delay. The main consideration in such a procedure is computational efficiency. In what follows we apply *Brent's Method* for the calculation of the optimal time delay. This a robust method which combines the Golden Section Search with Parabolic Interpolation, see [4] for details. In this paper we use the implementation available in Matlab's routine

fminbnd.

For Brent's Method it is necessary to establish an interval which brackets the optimal τ. The left (low) value τ_a can obviously be taken as the lossless τ, as can be seen in fig. 4. A suitable right value τ_b can be chosen by picking a time delay which produces a zero phase angle at some high frequency point. E.g., if one in the current examples specifies zero phase angle at 1 MHz, this gives a time delay of 84.58 µs.

A few remarks on choosing the frequency point for fixing τ_b is in order. In practice, one will normally specify a desired rms-error *tol* (e.g. 0.001) for the approximation. One should then identify the frequency point ω where the magnitude of H equals *tol*. The propagation velocity at this frequency equals

$$v = \frac{\omega}{\text{Im}\{\sqrt{Z(\omega) \cdot Y(\omega)}\}} \qquad (12)$$

which gives a time delay

$$\tau_b = \frac{l}{v} \qquad (13)$$

This approach for choosing τ_b is "safe" as it always gives an rms-error which is significantly higher than the minimum value.

B. Practical Implementation

It is necessary to write a routine which calculates the rational approximation and its rms-error for a given time delay. A skeleton program is shown in fig. 6. The effect of the time delay is first removed and the rational approximation and its rms-error is next calculated by VF. A few (three) iterations is used in order to ensure convergence of the pole identification.

$$[\text{rms}_{\text{err}}, \{R_m\}, \{a_m\}] = f_{calc}(H, s, \tau, N)$$

$$\boxed{\begin{aligned} &H'(s) = H(s) \cdot e^{s\tau} \\ &\{a_m\} = \text{logspace}(s(1), s(N_s), N) \quad \%\,\text{initial poles} \\ &\quad \text{for iter}=1{:}3 \\ &\qquad [\text{rms}_{\text{err}}, \{R_m\}, \{a_m\}] = VF(H', s, \{a_m\}) \\ &\quad \text{end} \end{aligned}}$$

Fig. 6. Skeleton program for calculating rms-error as function of time delay

C. Example

We continue with the same example and use the following bracketing values: [τ_a=83.33 µs, τ_b=85 µs]. With N=10, application of Brent's Method gave a minimum rms-error of 1.207E–4 at τ=83.93 µs. The following lists the output dialogue from the Matlab routine `fminbnd.m`. A tolereance of 1E–8 was specified for the convergence criterion.

Func-count	x	f(x)	Procedure
1	8.39699e-005	0.000174643	initial
2	8.43634e-005	0.0026967	golden
3	8.37268e-005	0.000323026	golden
4	8.3876e-005	0.000146437	parabolic
5	8.38984e-005	0.000131073	parabolic
6	8.39121e-005	0.000124677	parabolic
7	8.39342e-005	0.000120785	golden
8	8.39309e-005	0.000120687	parabolic
9	8.39275e-005	0.000120896	parabolic

Fig. 7. Output dialogue from fminbnd

Fig. 8 shows the magnitude of the (complex) fitting error of H when the time delay is taken as the lossless delay and the "optimal" delay, respectively. It can be seen that the optimal delay gives a significantly smaller deviation than the lossless delay, both in terms of rms-error and maximum error.

The total time consumption was <u>0.7 sec</u> on a PC with a 1.3 GHz Pentium processor. Note that computation involved a total of 9·3=27 calls to `vectfit.m`, with order N=10 and N_s=120 logaritmically spaced frequency samples.

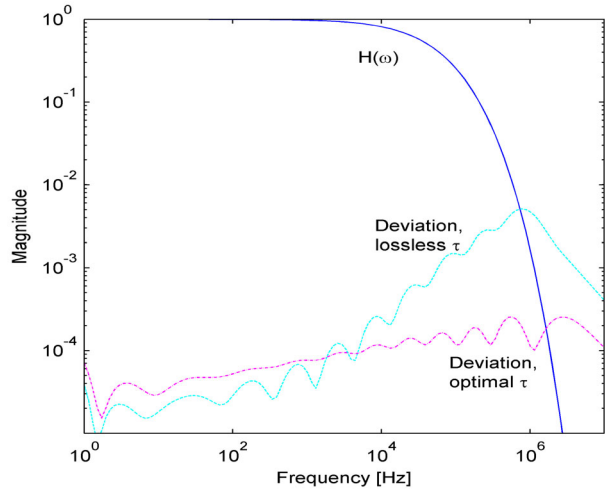

Fig. 8. Improvement by optimal time delay over lossless time delay (N=10).

D. User Interface

In practical simulation programs it is desirable that the user specifies the desired accuracy of the rational approximation, instead of the order of approximation. The program must then increment the order of approximation until the required accuracy is achieved.

Estimation of Lossess Time delay

In some situations it could be difficult to precalculate the lossless time delay τ_0, which is used as the left bracketing value τ_a in Brent's Method. Fortunately τ_0 can be estimated quite accurately via the magnitude of H by considering that multiplying H with $\exp(j\omega\tau_0)$ gives a minimum phase shift function. Bode [6] has shown that the phase angle of a minimum phase shift function can be calculated directly from the magnitude function [6]. The expression can be written [7]

$$\angle h_{\min}(\omega_l) = \frac{\pi}{2} \left. \frac{d(\ln|h(\omega)|)}{d(\ln\omega)} \right|_{\omega=\omega_l} + \Delta(u) \qquad (14)$$

where

$$\Delta(u) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\left| \frac{d(\ln|H(\omega)|)}{du} \right| - \left| \frac{d(\ln|H(\omega)|)}{du} \right|_{u=0} \right) \cdot \ln(\coth\frac{|u|}{2}) \, du \qquad (15)$$

and

$$u = \ln\frac{\omega}{\omega_l} \qquad (16)$$

The first term in (14) considers the derivative at the

considered frequency ω_1, whereas the second term takes into account the shape of $H(\omega)$.

Fig. 9 compares the the phase angle $\angle H_{min}(\omega)$ calculated by (14)-(16) with the phase angle of $H\cdot\exp(j\omega\tau_0)$. The very close agreement between the two phase angles shows that multiplication with $\exp(j\omega\tau_0)$ does indeed lead to a minimum-phase shift function. From the calculated phase angle H_{min} at some frequency $\omega=\omega_1$, the time delay can be calculated as [8]:

$$\tau_{min} = \frac{l}{v(\omega_1)} + \frac{\angle h_{min}(\omega_1)}{\omega_1} \qquad (17)$$

Note that the integral in (15) should be taken over a frequency range that spans at least two decades below and above ω_1, i.e. $\omega_{low}=0.01\omega_1$, $\omega_{high}=100\omega_1$. The Appendix shows a matlab code for calculating (14)-(16).
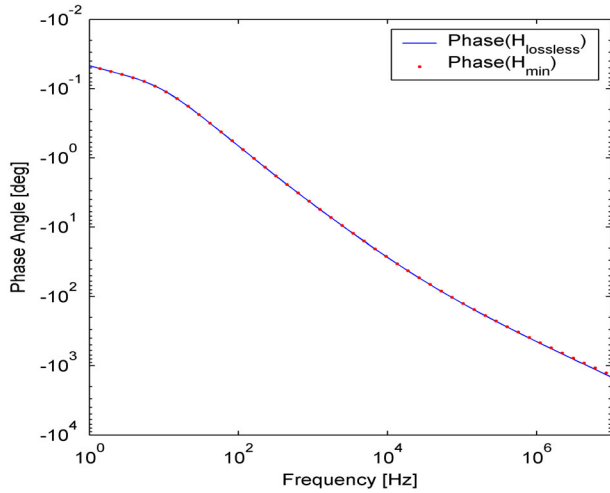

Fig. 9. Phase angle of $H\cdot\exp(j\omega\tau_0)$ and of minimum-phase shift function

Multi-conductor systems

The previous sections have shown how to approximate the propagation function H of a single conductor system with rational functions plus a time delay. The application to multi-conductor systems is discussed next.

Some multi-conductor models rely on diagonalization by a constant, real transformation matrix [1],[2]. This modeling technique is perfectly valid for balanced (continuously transposed) lines, and it usually gives sufficiently accurate results for unbalanced lines. The modeling technique leads to a set of decoupled modes which are represented by uncoupled single conductor line models. Each mode of H needs to be fitted with a rational function plus a time delay. The computational procedure described in this paper is directly applicable to the modes of balanced lines. It also applies to unbalanced lines provided provided that the transformation matrix is calculated at a sufficiently high frequency.

Reference [3] describes a highly successful line model for multi-conductor systems which is based on identifying poles and time delays from modes, followed by residue identification in phase coordinates. The practical implementation of this line model in commercial programs (EMTDC, EMTP-RV) has adopted a simplified version of the time delay identification procedure described in this paper.

Conclusions

1. Assuming a lossless time delay will for a given order of approximation result in an unnecessary high fitting error.
2. Combining Vector Fitting with Brent's Method allows to optimize the time delay together with the poles and residues. This gives a reduced fitting error in terms of rms-error and maximum error.
3. The left bracketing time delay needed for Brent's Method can be taken as the lossless time delay. A procedure for determining the right bracketing value is given in the paper.
4. The lossless time delay can alternatively be calculated via the magnitude function assuming a minimum phase-shift function.
5. The procedure is applicable to single conductor systems and to the modes of multi-conductor systems.

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Appendix — Computation of Phase Angle

The following shows a Matlab code for calculating the phase angle of a minimum-phase shift function by (14)-(16) and the time delay by (17). It is assumed that absH contains the magnitude function, given at Ns frequency samples. The code calculates the phase angle at the jth frequency sample.

```
%First term in (14):
phase1=(pi/2)*log((absH(j+1)/absH(j-1)))/…
       (log(w(j+1)/w(j-1)));

%Second term in (14):
phase2=0;
for k=1:Ns-1
  term1=log((absH(k+1)/absH(k)))/(log(w(k+1)/w(k)));
  term2=log((absH(j+1)/absH(j-1)))/…
        (log(w(j+1)/w(j-1)));
  phase2=phase2+(abs(term1)-abs(term2))*…
         log(coth(abs(log((w(k)+w(k+1))/…
         (2*w(j))))/2 )) *log(w(k+1)/w(k));
end
phase2=phase2/pi;
phase_min=(phase1-phase2); %Phase angle [rad]
tau_min  =line_length/veloc(j)+phase_min/w(j); %[sec]
```