A FRONT-TRACKING METHOD FOR HYPERBOLIC THREE-PHASE MODELS

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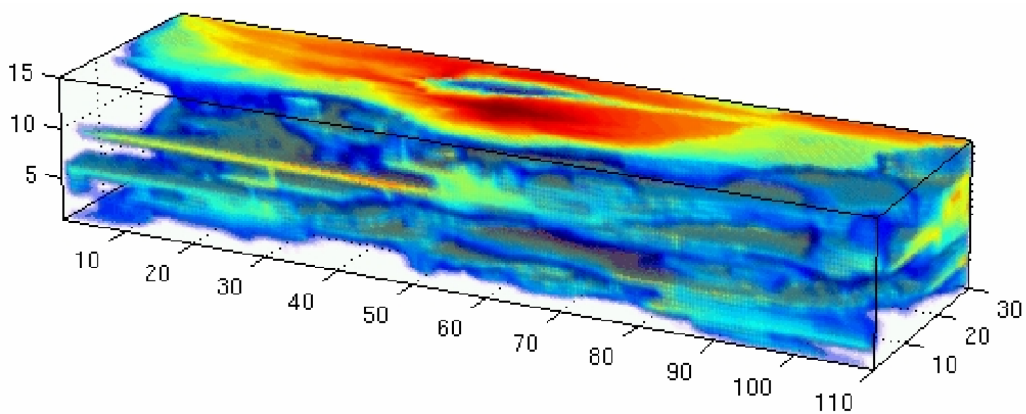

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WHAT DO WE PROPOSE, AND WHY?

- **Objective:**
  - Exceptionally accurate, fast numerical solutions to realistic **three-phase flows** in porous media
- **Approach:**
  - Develop **analytical solution** to the Riemann problem
  - Use it as a building block for general 1D problems, via a **front-tracking** method
  - Solve three-phase flow along **streamlines**

MATHEMATICAL MODEL

- **Assumptions:**
  - Immiscible, incompressible fluids
  - Multiphase extension of Darcy's law
  - Negligible capillary effects

- **Equations:**
  - *Pressure* equation (elliptic)

$$\nabla \cdot \mathbf{v}_T = 0, \qquad \mathbf{v}_T = -\lambda_T \frac{\mathbf{k}}{\phi} \nabla p, \qquad \lambda_T \equiv \lambda_w + \lambda_o + \lambda_g$$

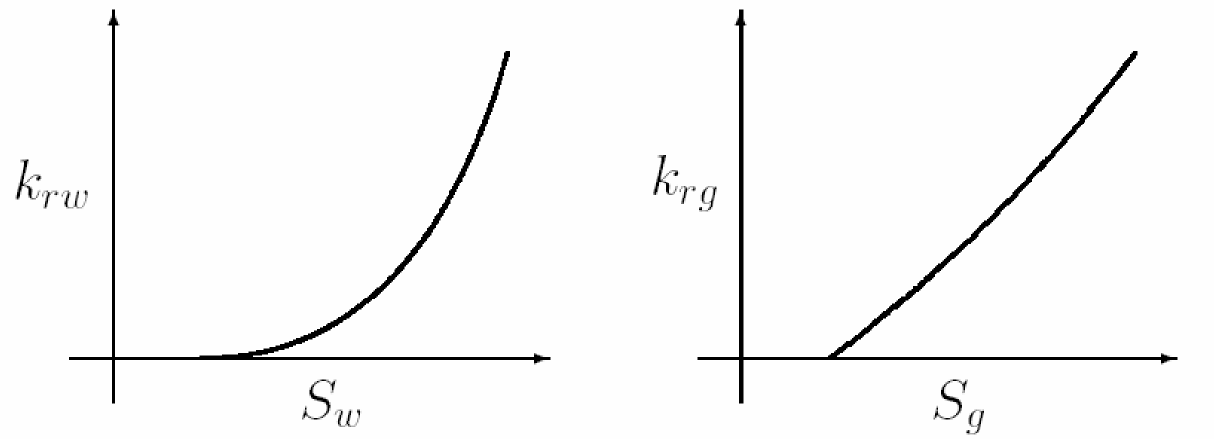
  - A system of *saturation equations* (hyperbolic)

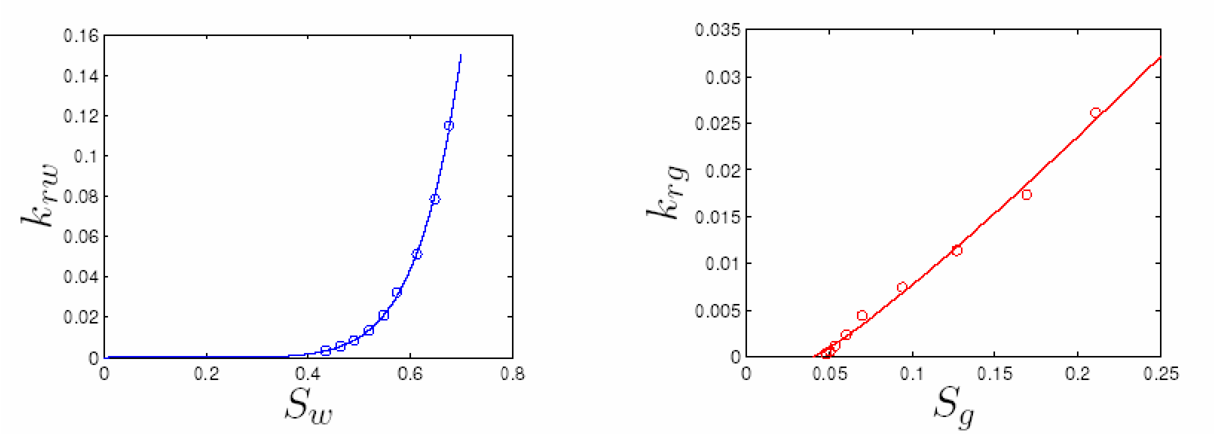
$$\partial_t \begin{pmatrix} S_w \\ S_g \end{pmatrix} + \mathbf{v}_T \cdot \nabla \begin{pmatrix} f_w \\ f_g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad f_w = \lambda_w / \lambda_T, \qquad f_g = \lambda_g / \lambda_T$$

CONDITIONS FOR HYPERBOLICITY

(J. and Patzek: *TIPM* in press)

- **Essential condition:** a positive endpoint slope of the relative permeability of the least wetting phase



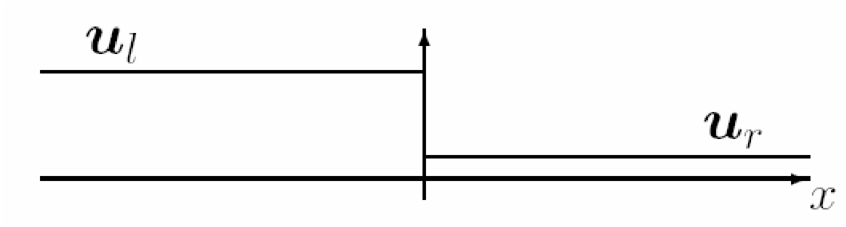


THE THREE-PHASE RIEMANN PROBLEM

- **Riemann problem:** find a weak (possibly discontinuous) solution to the 2×2 system of equations

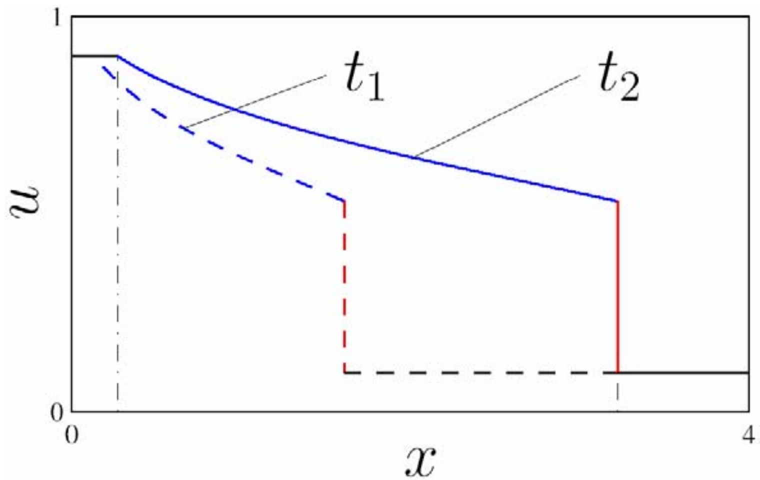
$$\partial_t \mathbf{u} + \partial_x \mathbf{f} = \mathbf{0}, \quad -\infty < x < \infty,\ t > 0$$

$$\mathbf{u}(x,0) = \begin{cases} \mathbf{u}_l, & x < 0 \\ \mathbf{u}_r, & x \geq 0 \end{cases}$$

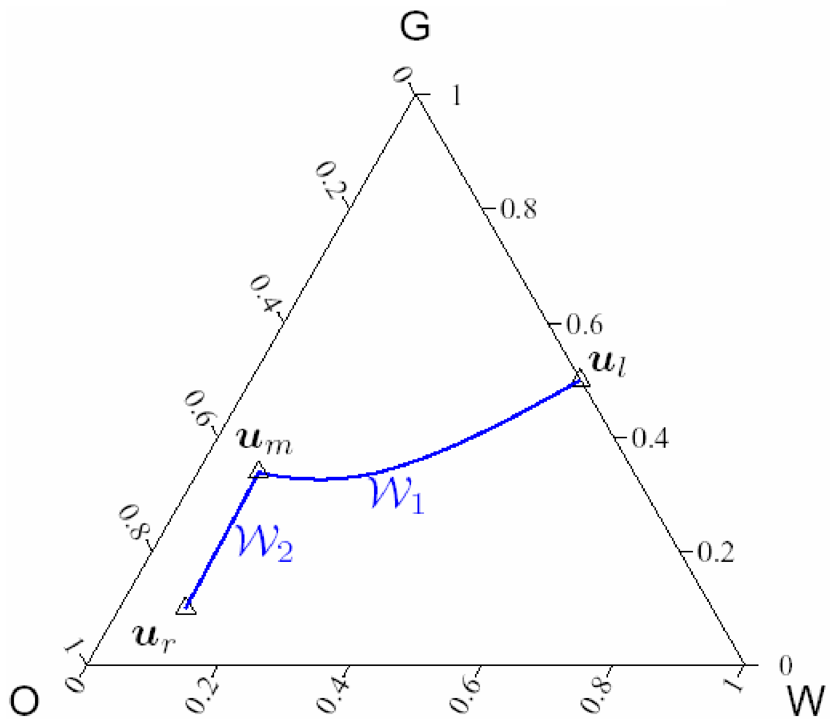


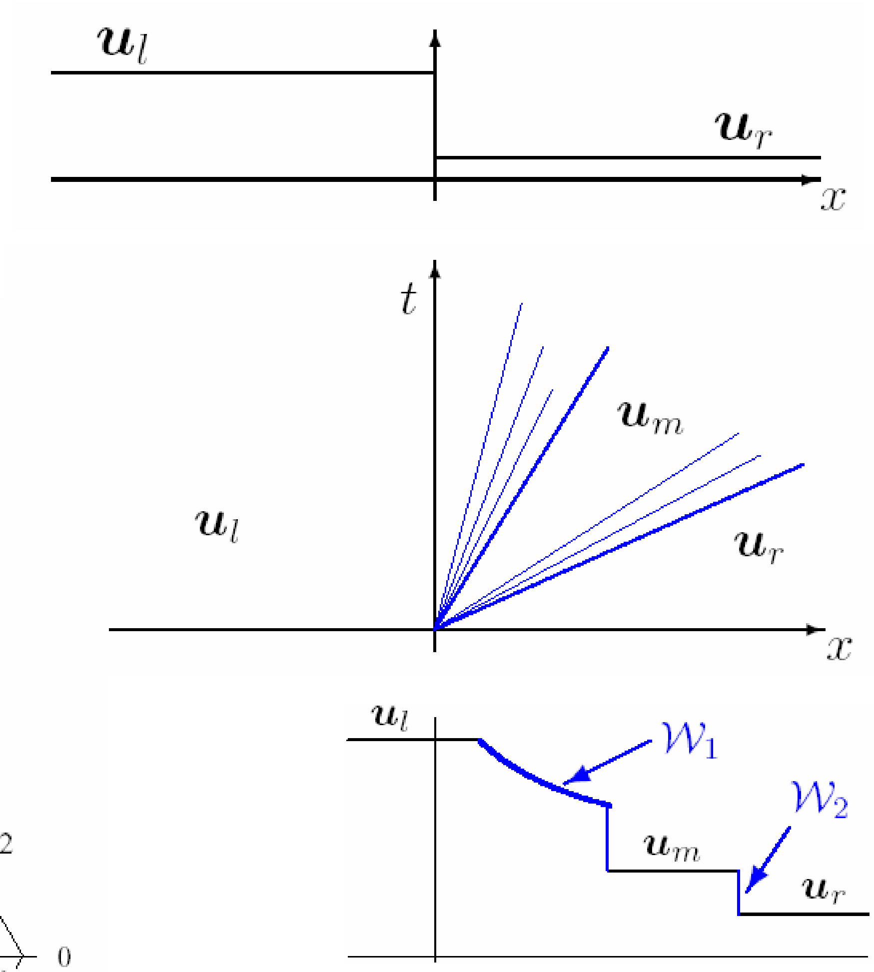
- **Self-similarity** (“stretching” or “coherence” principle):

$$\mathbf{u}(x,t) = \mathbf{U}(\varsigma), \text{where } \varsigma = x/t$$



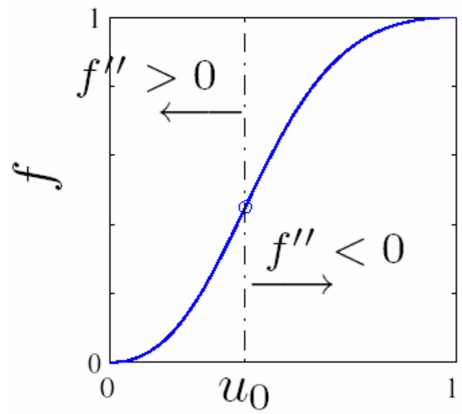
SOLUTION OF THE RIEMANN PROBLEM



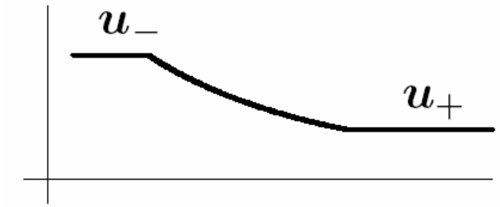


WAVE TYPES (TWO-PHASE FLOW)

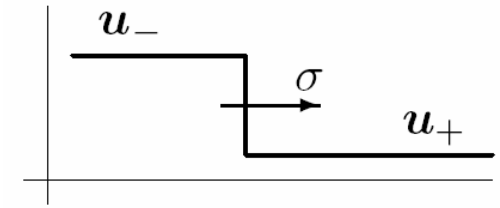
- The fractional flow function is S-shaped, with a single inflection point



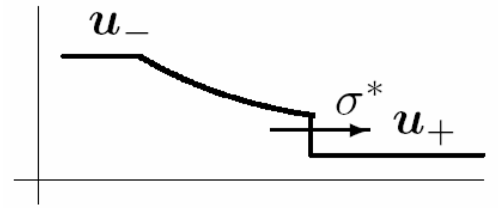
- The only **admissible wave types** are:
  - Rarefaction (R)



  - Shock (S)

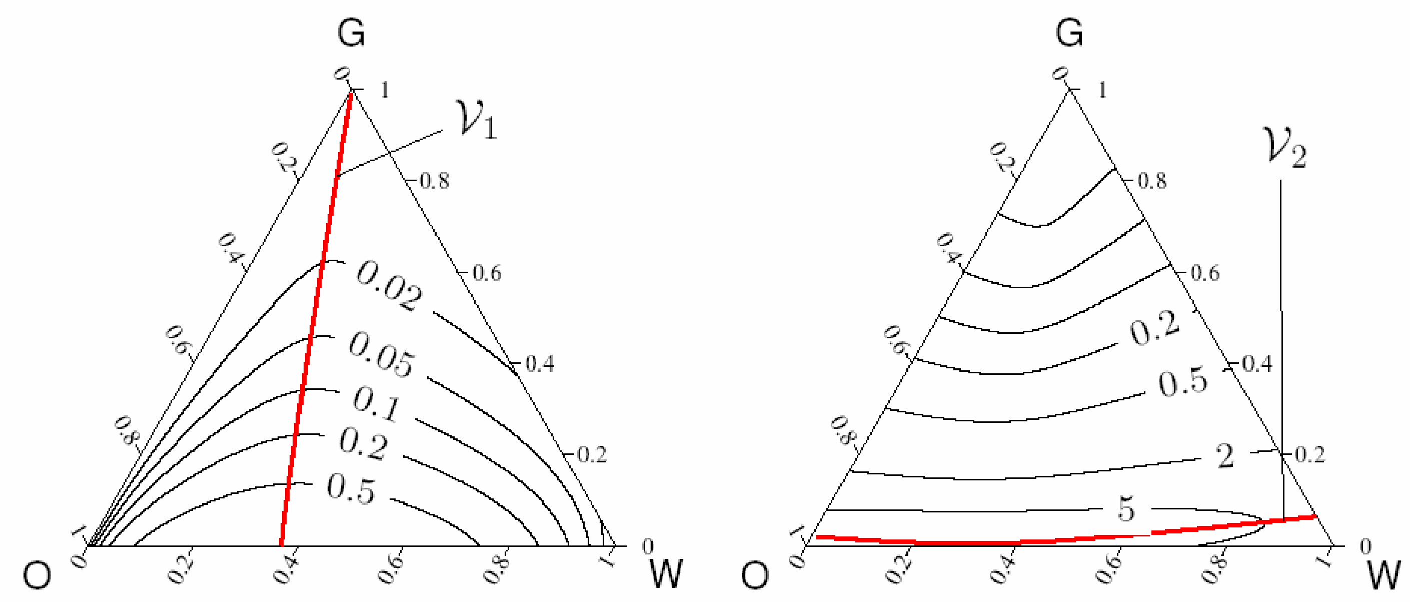


  - Rarefaction-shock (RS)



WAVE TYPES (THREE-PHASE FLOW)

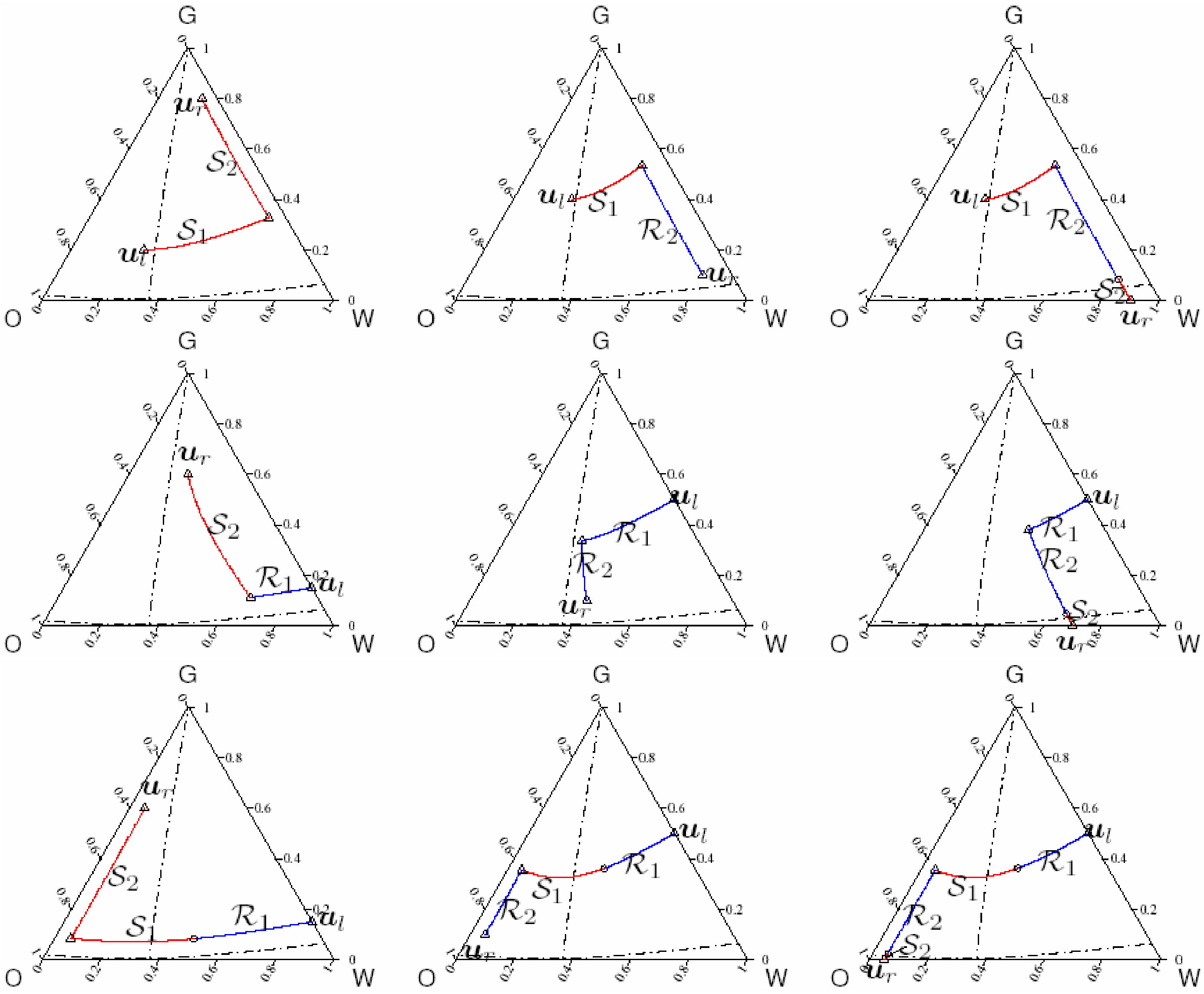
- The fractional flow functions have single, continuous inflection loci (natural generalization of the two-phase case)



- There are **9 admissible wave combinations**
  - Two separate waves: W_1, W_2
  - Each wave may only be of type R, S, or RS

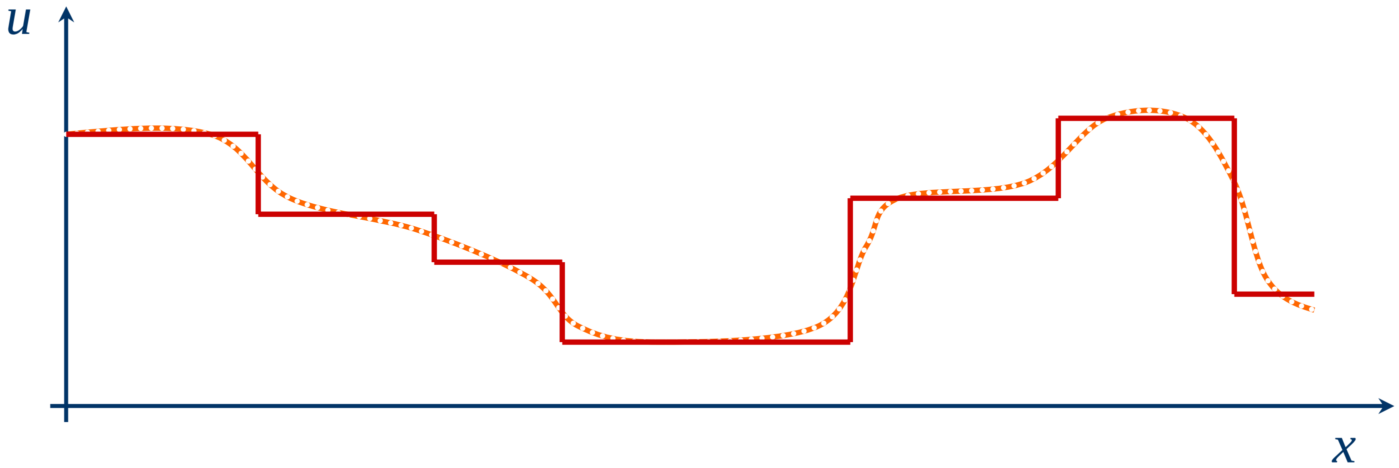
WAVE TYPES (THREE-PHASE FLOW)

(J. and Patzek: *TIPM* 2004)



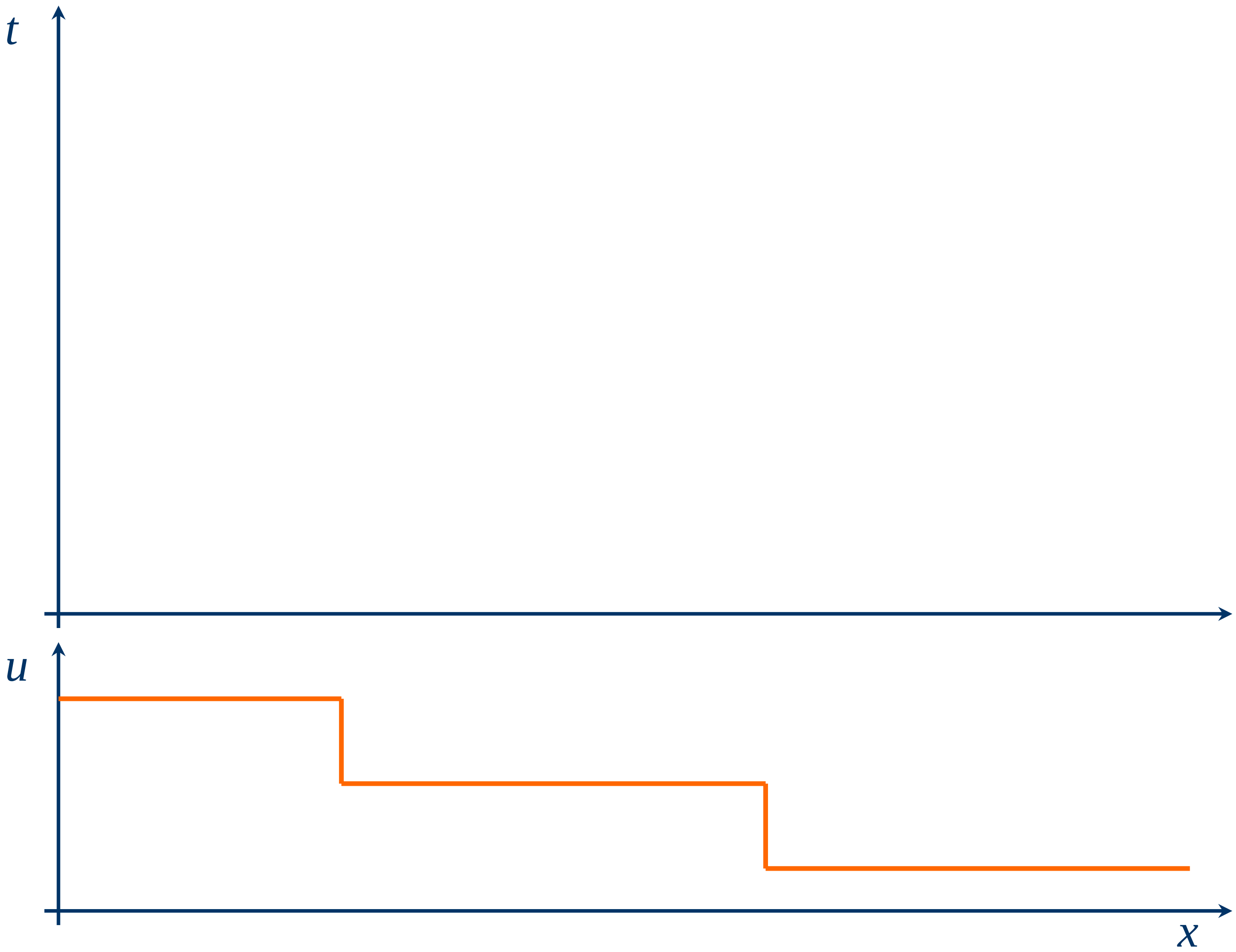
THREE-PHASE CAUCHY PROBLEM

- Solution to the Riemann problem is insufficient if:
  - Initial conditions different from constant
  - Variable injection saturations (e.g. WAG)

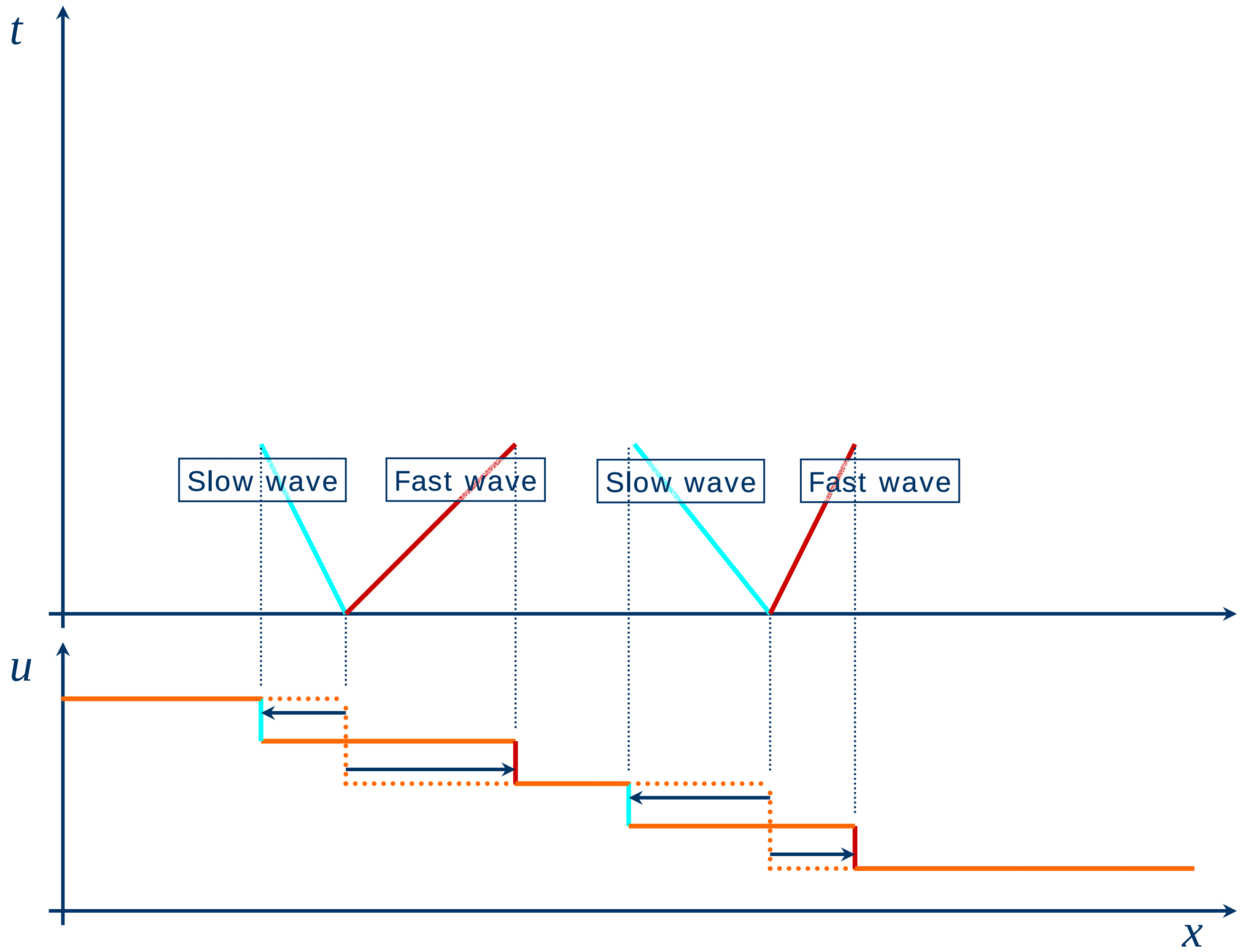


- **Front-tracking method:**
  - Piecewise constant approximation of the solution
  - Sequence of Riemann problems

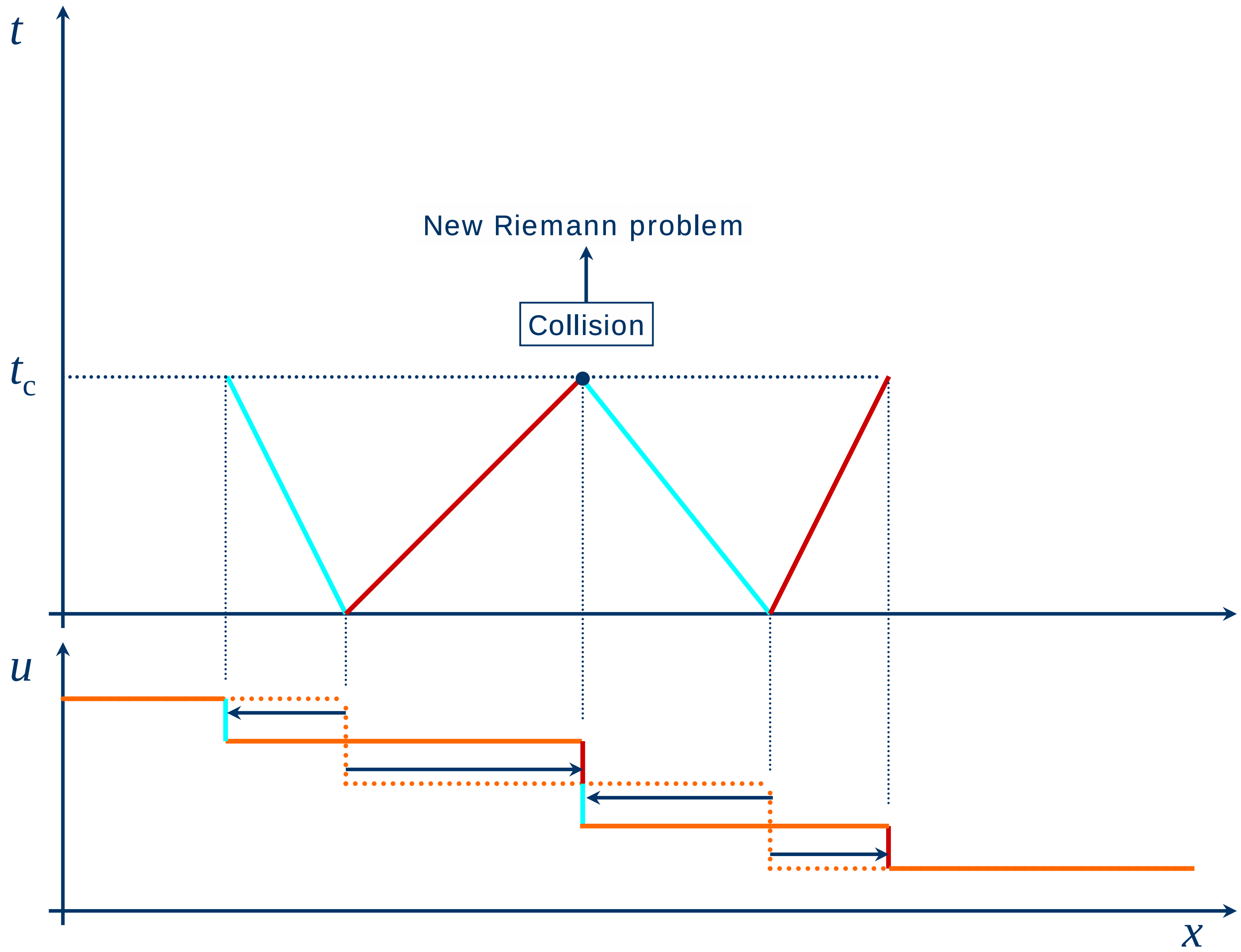
FRONT-TRACKING ALGORITHM



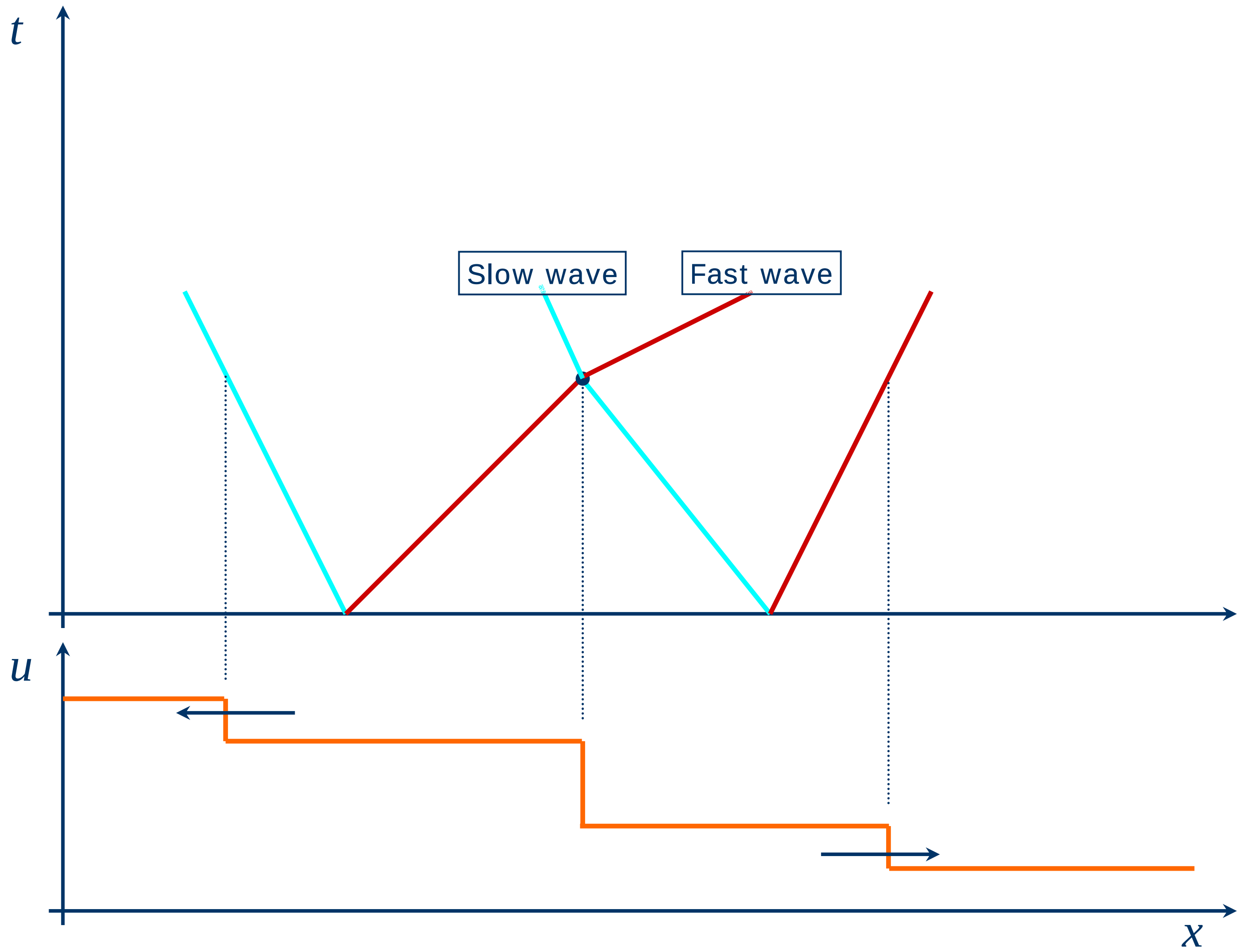
FRONT-TRACKING ALGORITHM



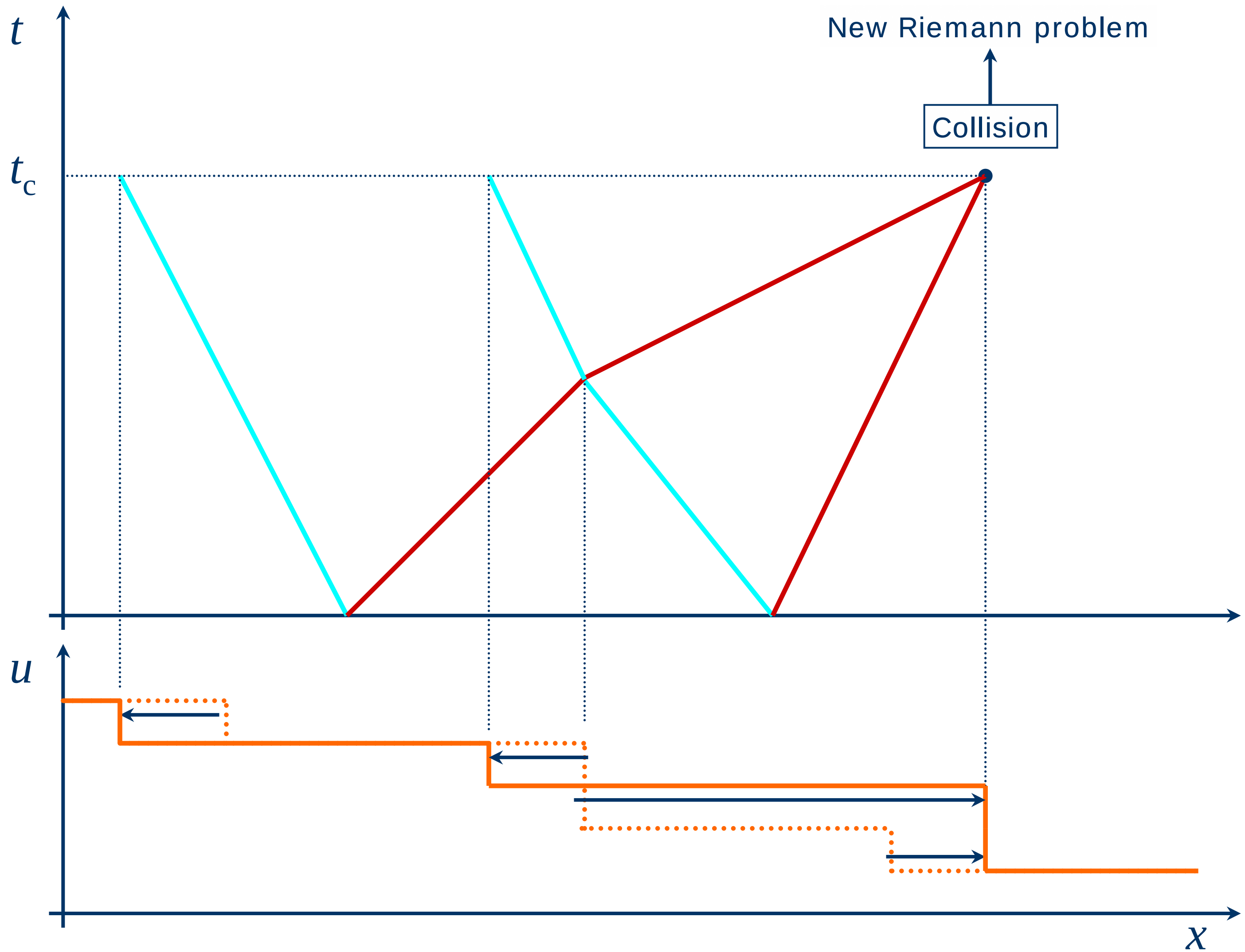
FRONT-TRACKING ALGORITHM



FRONT-TRACKING ALGORITHM

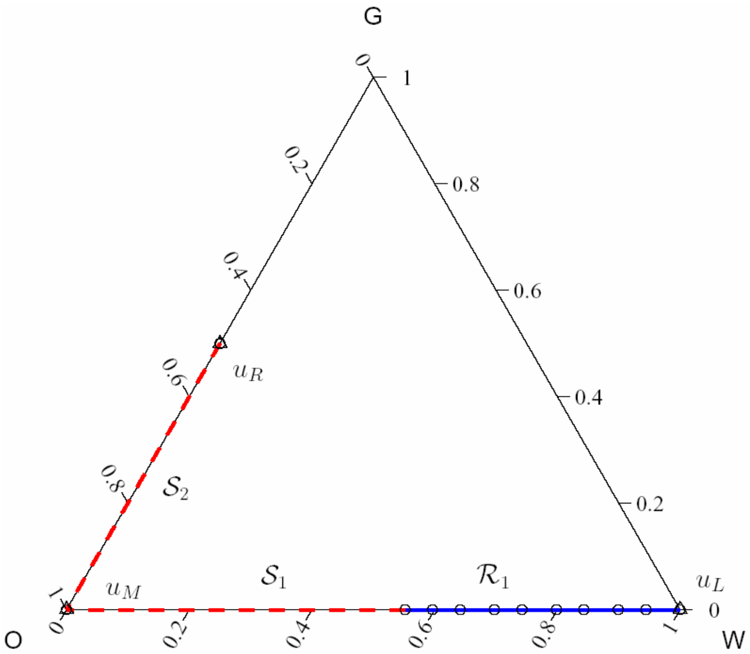


FRONT-TRACKING ALGORITHM

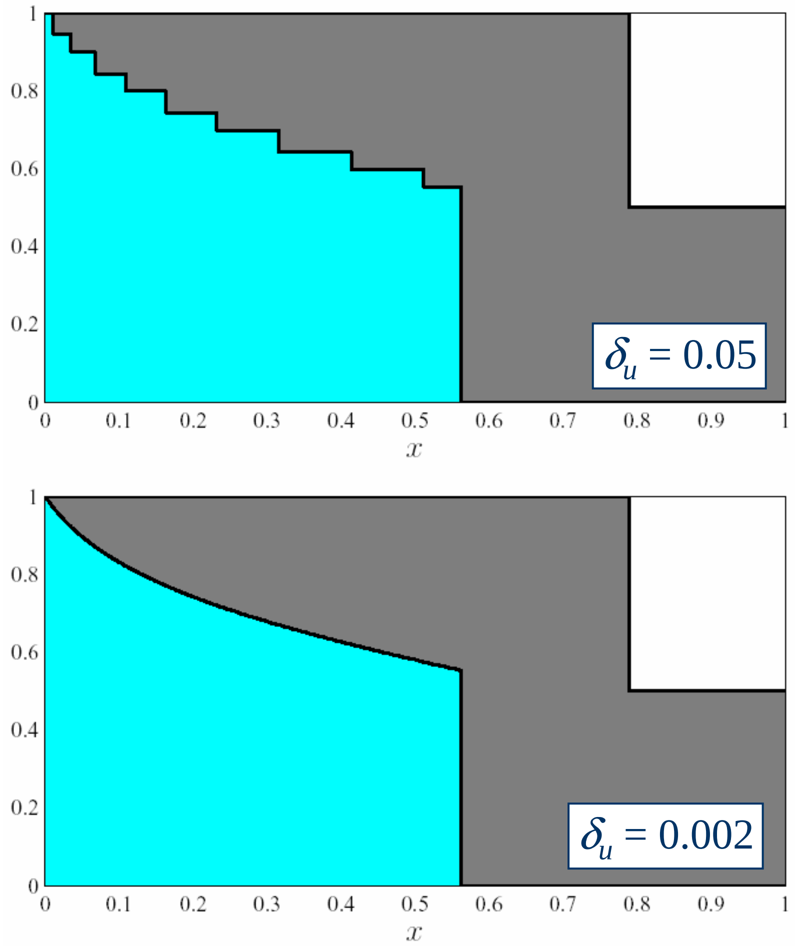


EXAMPLE 1

- Riemann problem involving **local** wave curves



Saturation path:
Exact solution and front-tracking solution



EXAMPLE 2: LINEAR WAG

- Initially, reservoir with 80% oil, 20% gas
- Alternate cycles of water and gas injection
- Front-tracking solution with d_u = 0.005
- Half a million Riemann solves ~ 5 sec on a desktop PC

STREAMLINE METHODS

- **Basic idea:** decouple the three-dimensional transport into a series of 1D problems along streamlines

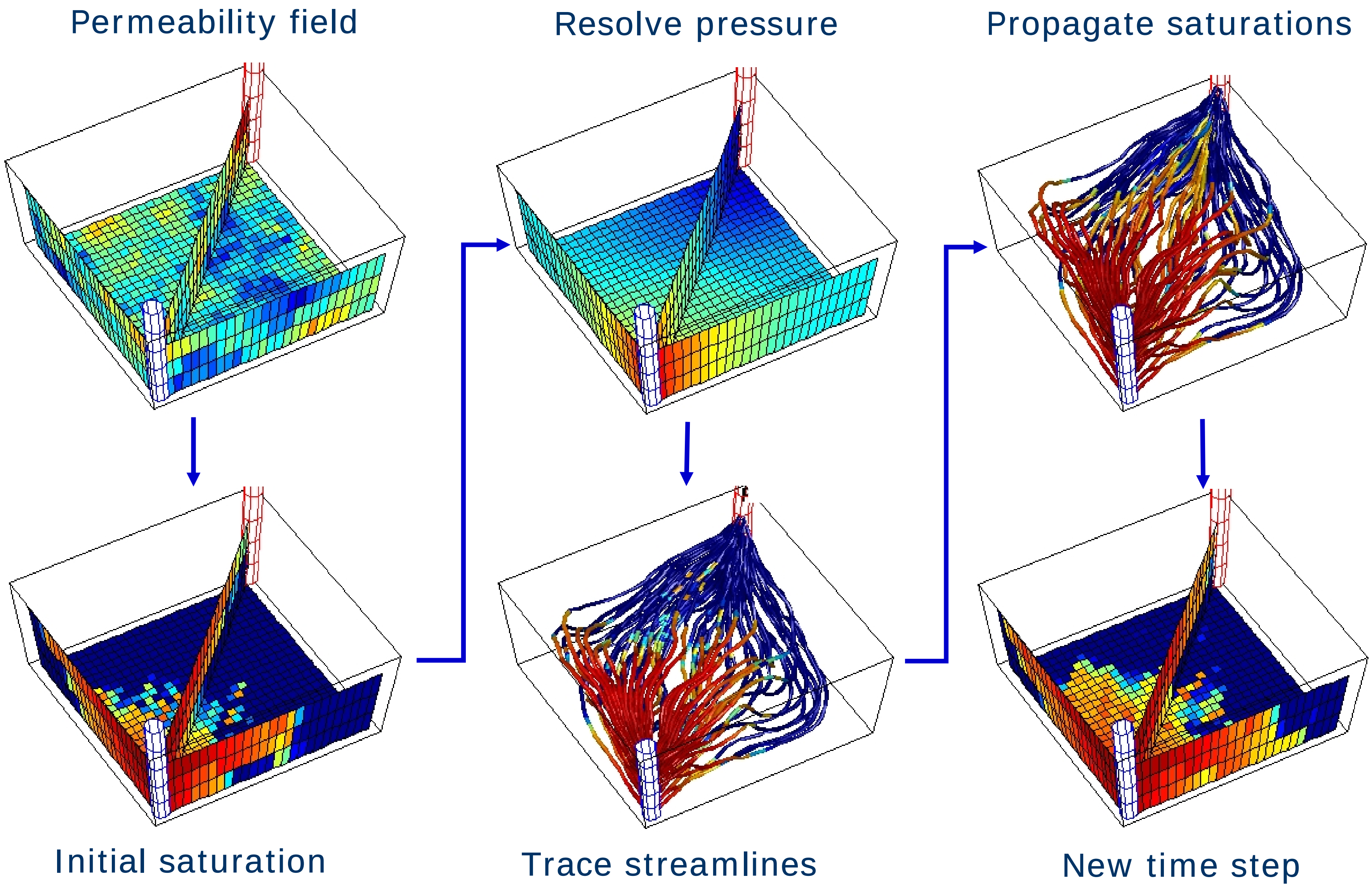
- **Sequential solution** of pressure and saturations (IMPES)
  - Pressure equation (fixed saturations)

$$\nabla \cdot \mathbf{v}_T = 0, \quad \mathbf{v}_T = -\lambda_T \frac{\mathbf{k}}{\phi} \nabla p$$

  - Compute streamlines for the velocity field $\mathbf{v}_T$
  - System of saturation equations (along each streamline)

$$\partial_t \begin{pmatrix} S_w \\ S_g \end{pmatrix} + \partial_\tau \begin{pmatrix} f_w \\ f_g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \text{where } \tau(s) \equiv \int_0^s \frac{1}{|v_T|} \mathrm{d}\xi$$

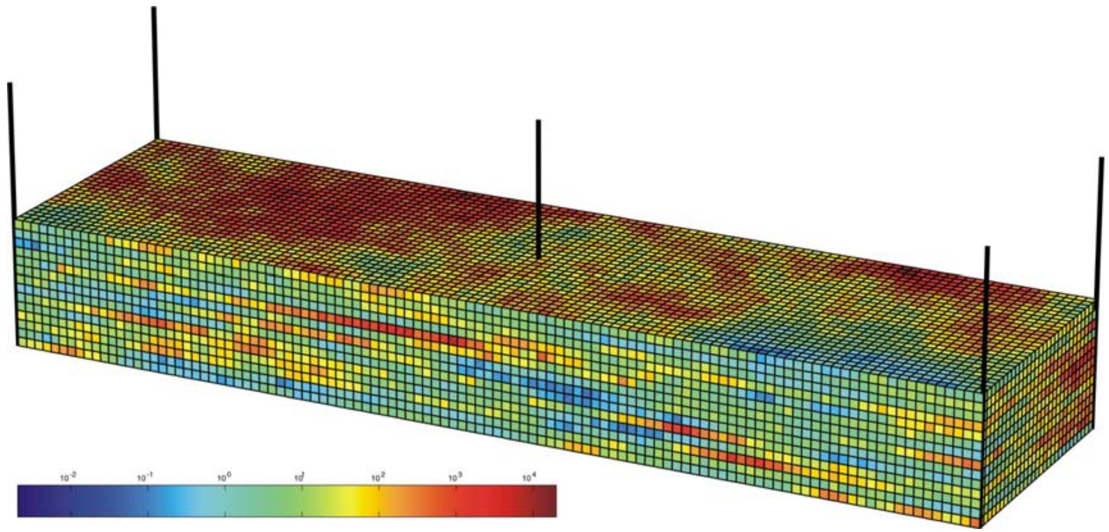
(Figures from Yann Gautier)



NUMERICAL SIMULATIONS

(Lie and J.: *CGEOS* submitted)

- Highly heterogeneous, shallow-marine formation, taken from the **SPE10 comparative solution project**
  - Permeability variations of 6 orders of magnitude
  - Five vertical wells (1 injector, 4 producers)



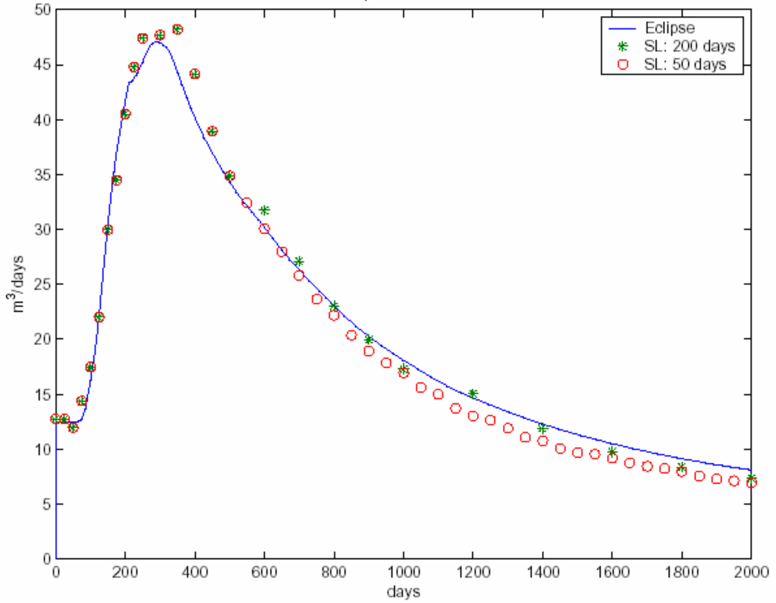
  - Two different injection schemes:
    (1) Continuous water injection
    (2) Water-alternating-gas injection (WAG)

FLUID PRODUCTION

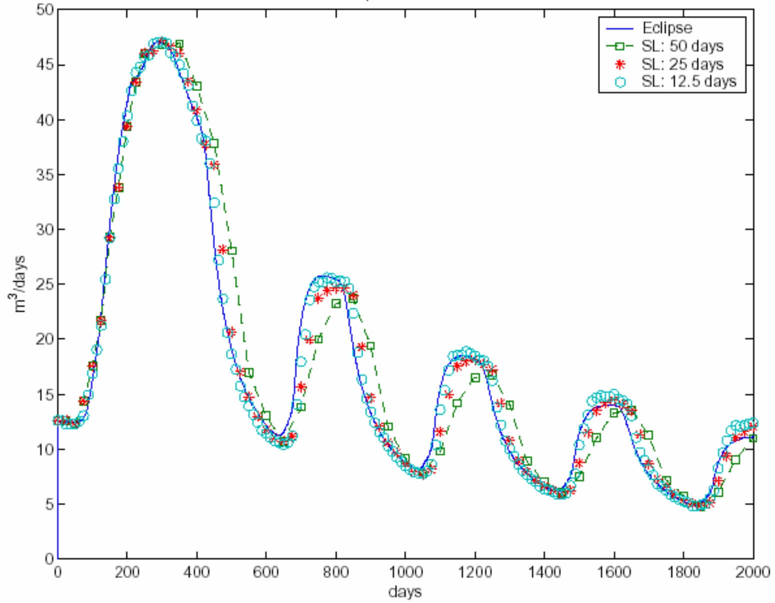
- Comparison of fluid recovery predictions against the commercial reservoir simulator Eclipse® (Schlumberger)

- **Oil production rate**

Continuous water injection

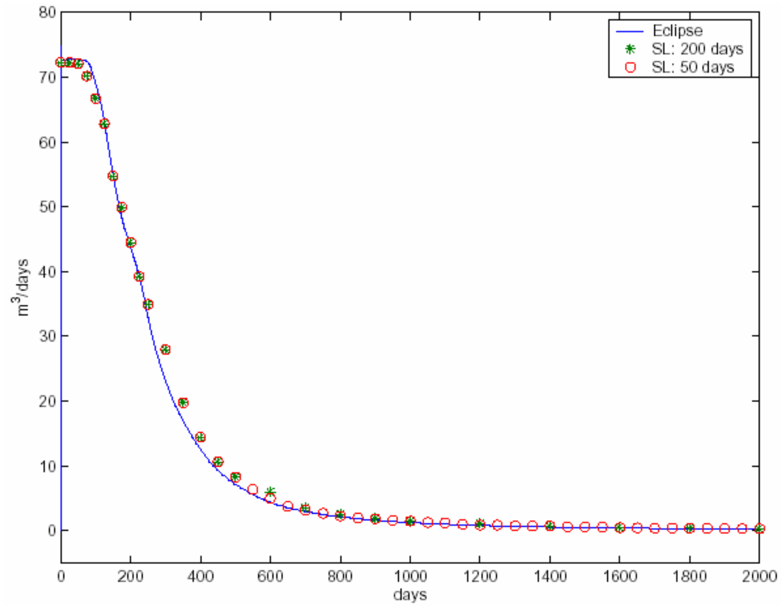


Water-alternating-gas injection

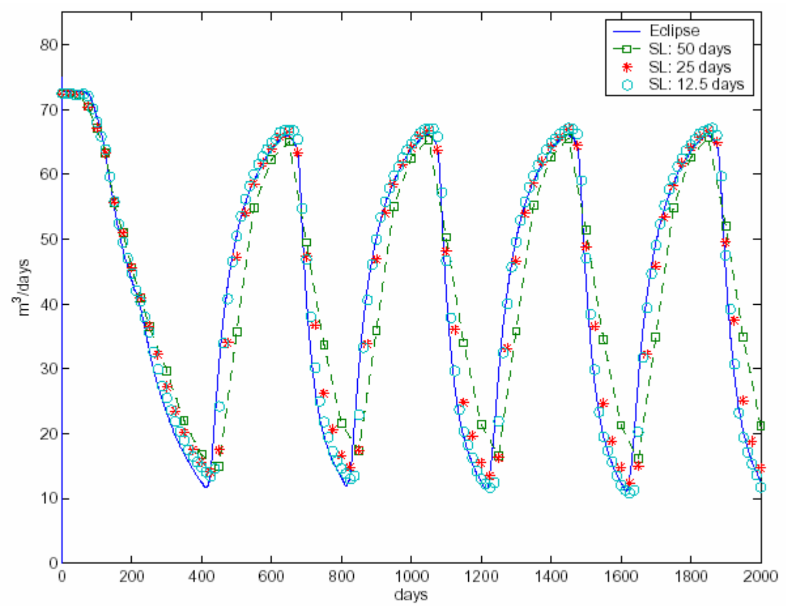


■ Gas production rate

Continuous water injection

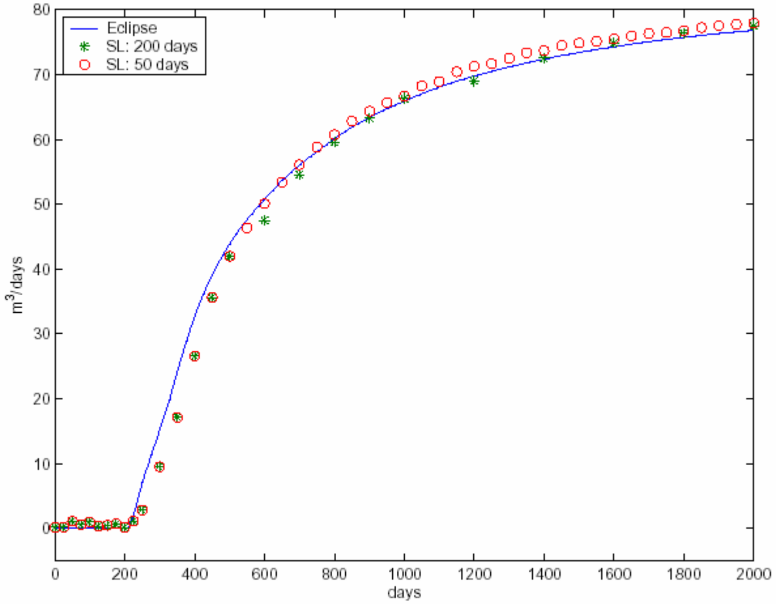


Water-alternating-gas injection

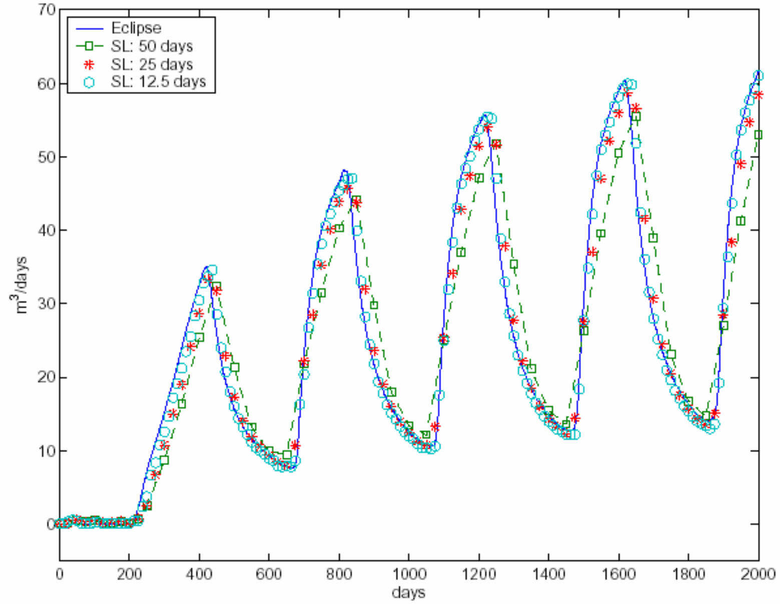


■ Water production rate

Continuous water injection



Water-alternating-gas injection



■ CPU times:

	Water injection	WAG
ECLIPSE	1h 22min	8h 20min
Streamline	50min (d*t* = 200 days)	2h 13min (d*t* = 25 days)

CONCLUSIONS

- The integration of **analytical Riemann solvers**, the front-tracking method, and streamline simulation, offers the potential for fast and accurate prediction of three-phase flow in highly-heterogeneous reservoirs

FUTURE WORK

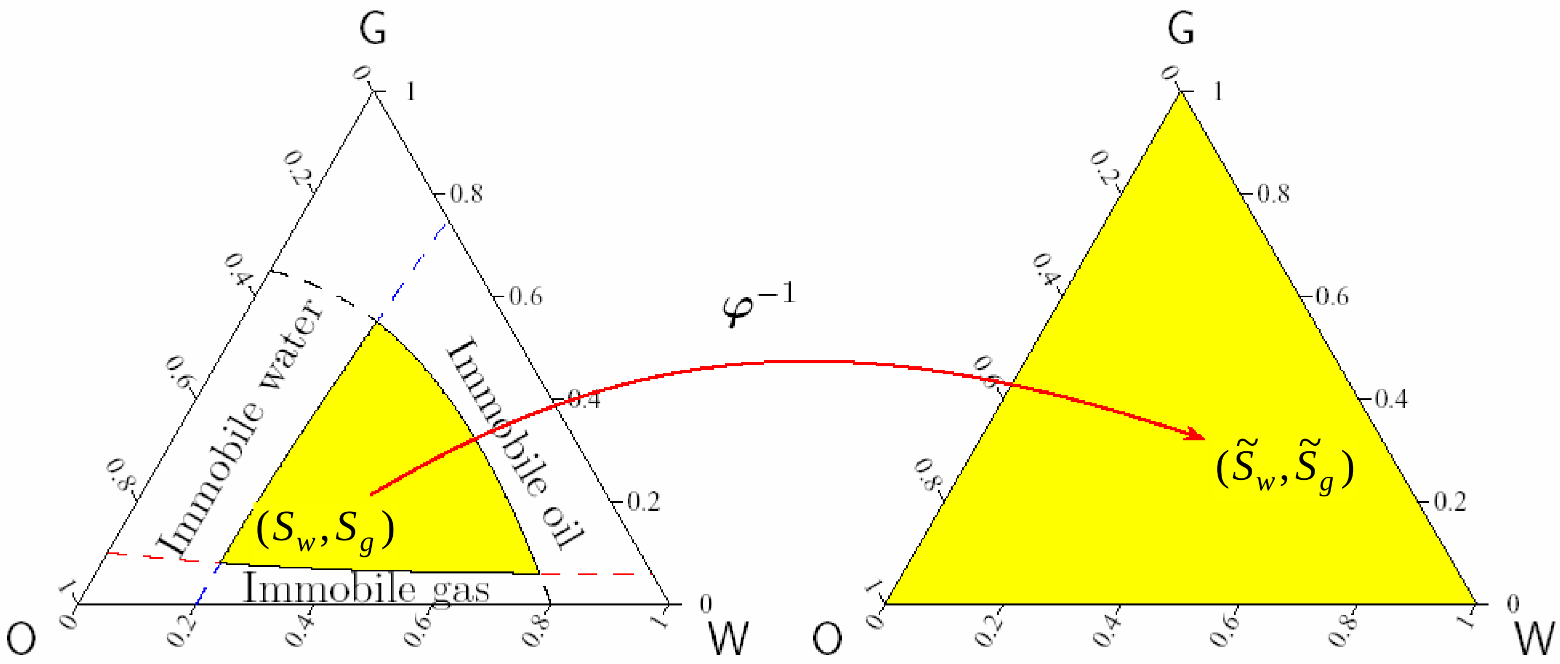
- **Extend the Riemann solver**
  - Residual saturations
  - Relative permeability hysteresis
  - Fluid miscibility and compositional effects
- **Extend the streamline simulator**
  - Gravity, compressibility, and capillary pressure effects

PUBLICATIONS

- R. Juanes. *Displacement Theory and Multiscale Numerical Modeling of Three-Phase Flow.* PhD Dissertation, University of California, Berkeley, 2003.
- R. Juanes, T.W. Patzek. Relative permeabilities for strictly hyperbolic models of three-phase flow in porous media. *Transport in Porous Media* (accepted, in press).
- R. Juanes, T.W. Patzek. Analytical solution to the Riemann problem of three-phase flow in porous media. *Transport in Porous Media*, **55**(1):47-70, 2004.
- R. Juanes, T.W. Patzek. Three-phase displacement theory: an improved description of relative permeabilities. *SPE Journal* (accepted, in press).
- R. Juanes, K.-A. Lie, V. Kippe. A front-tracking method for hyperbolic three-phase models. In *Proceedings* of ECMOR IX, Cannes, France, 2004.
- K.-A. Lie, R. Juanes. A front-tracking method for the simulation of three-phase flow in porous media. *Computational Geosciences* (in review).

Backup foils

THE SATURATION SPACE



CHARACTER OF THE SYSTEM

The character of the system of first-order equations

$$\partial_t \begin{pmatrix} u \\ v \end{pmatrix} + \partial_x \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \Leftrightarrow \quad \partial_t \mathbf{u} + \partial_x \mathbf{f} = \mathbf{0}$$

is determined by the eigenvalues (n_1, n_2) and eigenvectors ($\boldsymbol{r}_1$, $\boldsymbol{r}_2$) of the Jacobian matrix

$$\mathbf{f}'(\mathbf{u}) = \begin{pmatrix} f_{,u} & f_{,v} \\ g_{,u} & g_{,v} \end{pmatrix}$$

- **Hyperbolic:** the eigenvalues are real and
  the Jacobian matrix is diagonalizable
  - **Strictly hyperbolic:** eigenvalues are distinct, $n_1 < n_2$
- **Elliptic:** eigenvalues are complex conjugates

CONDITIONS FOR HYPERBOLICITY

- **Traditional approach:**

Assume certain behavior of the relative permeabilities → *Infer* loss of hyperbolicity

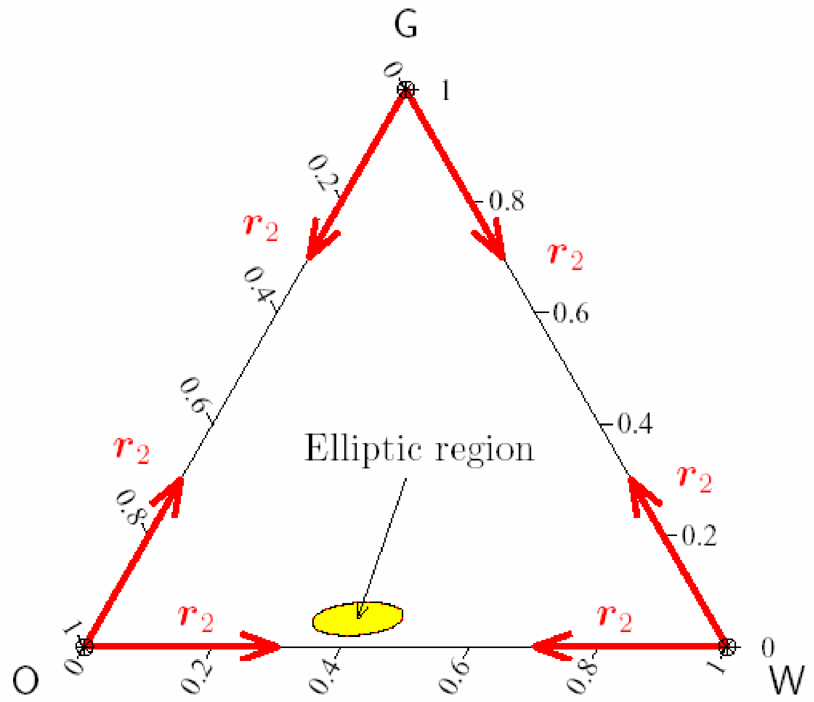
- **We use a new approach:**

(J.: *PhD* 2003)
(J. and Patzek: *SPEJ* in press)

Infer conditions on the relative permeabilities ← *Enforce* hyperbolicity

ELLIPTIC REGIONS

Regions in the saturation triangle, where the system of equations is **elliptic** rather than **hyperbolic**



RIEMANN SOLVER ALGORITHM

1. Given injected (left) and initial (right) states: u_L, u_R

2. Set initial guess and trial solution: u_M^{tr}, $W_1^{tr} = R_1$, $W_2^{tr} = R_2$

3. Solve trial configuration and update wave structure:

$$[u_M, W_1, W_2] = \text{WaveStruct}\,(u_L, u_R, u_M^{tr}, W_1^{tr}, W_2^{tr})$$

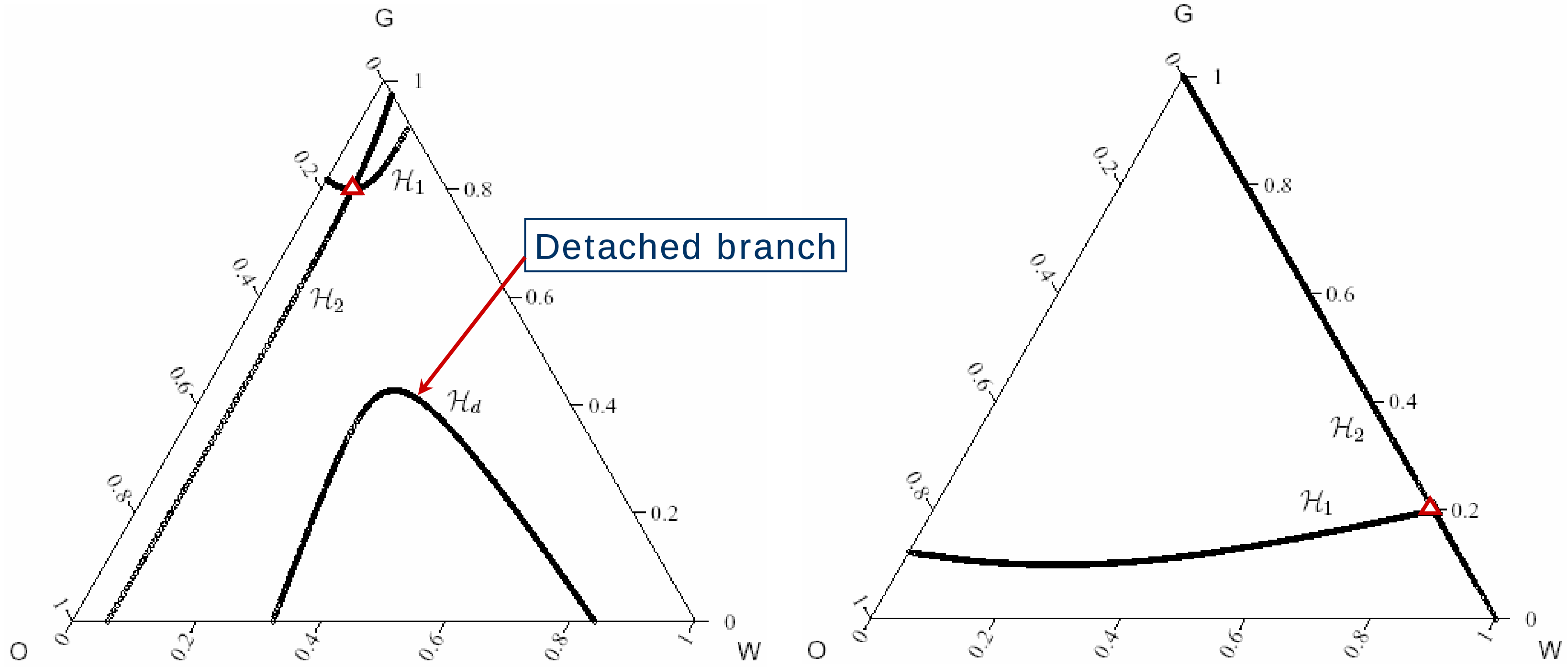
(J. and Patzek: *TIPM* 2004)

4. Check admissibility:
   If $(s_1 > s_2)$ { Set new initial guess: u_M
   Declare solution invalid: $W_1^{tr} = W_2^{tr} = 0$ }

5. Check convergence:
   If $W_1 W_2 = W_1^{tr} W_2^{tr}$ Stop
   Else Set $W_1^{tr} W_2^{tr} \leftarrow W_1 W_2$, $u_M^{tr} \leftarrow u_M$, Goto 3.

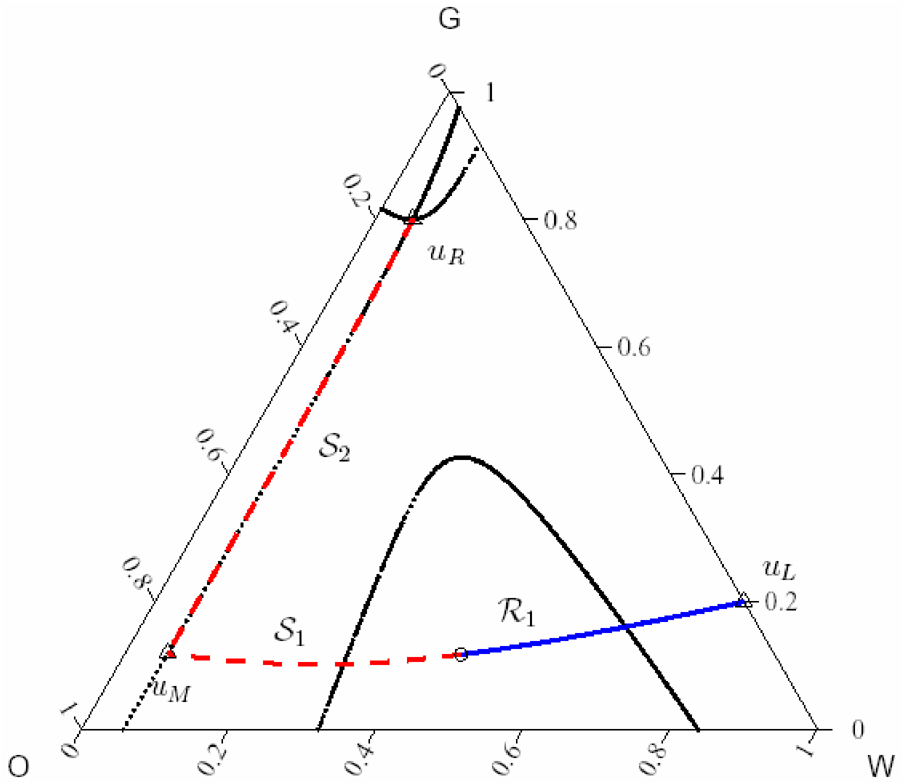
NONLOCAL WAVE CURVES

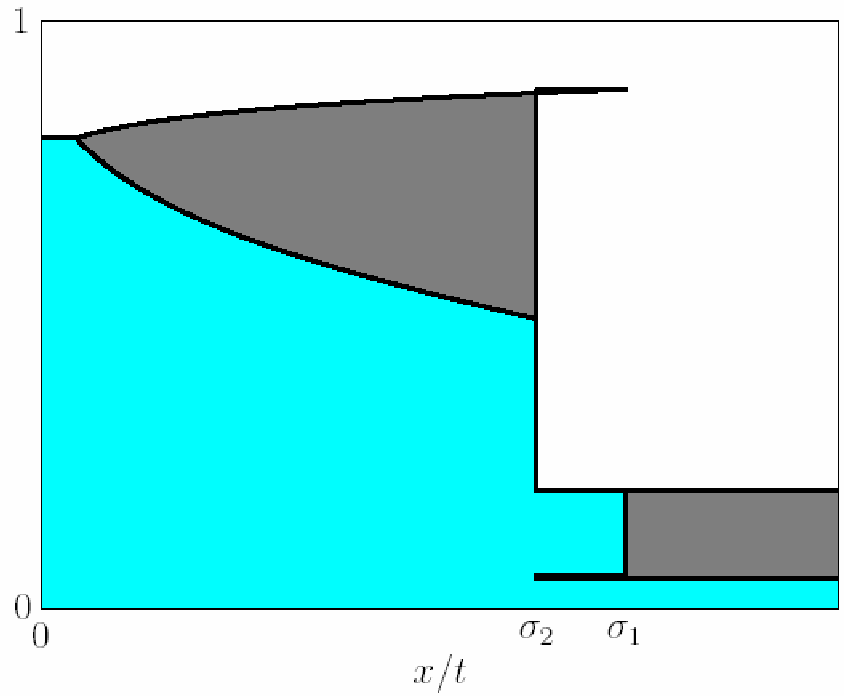
- Usual construction assumes that wave curves are *local*
- This construction may be globally inadmissible: $s_1 \not< s_2$
- Reason: shock curves may present **detached branches**



ROLE OF DETACHED BRANCHES

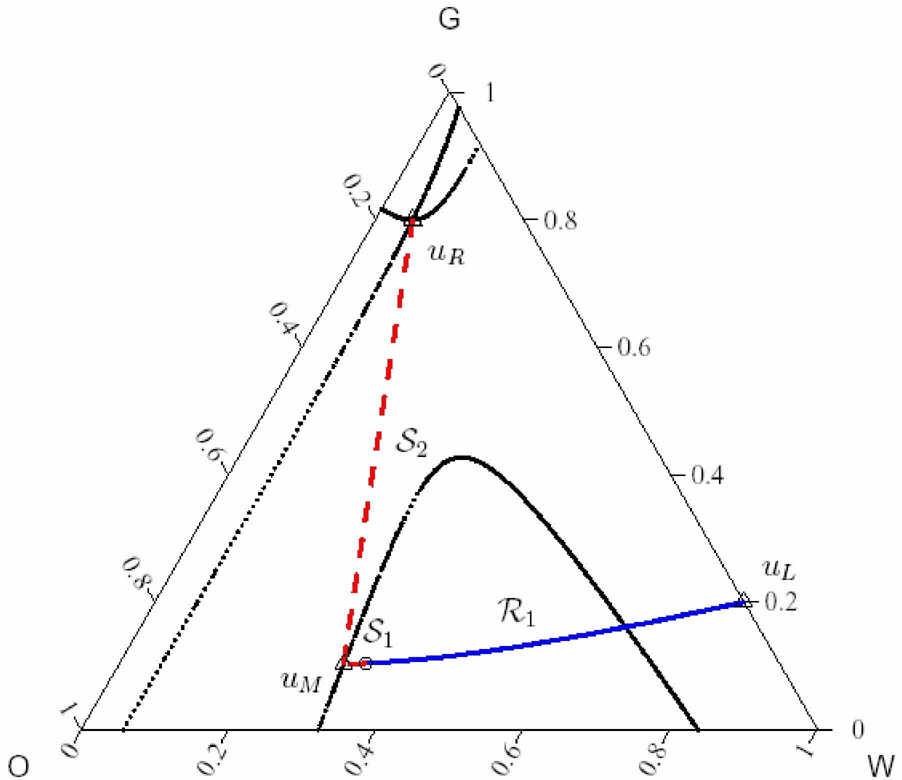
- **Inadmissible** solution involving **local** wave curves: $s_1 > s_2^{loc}$

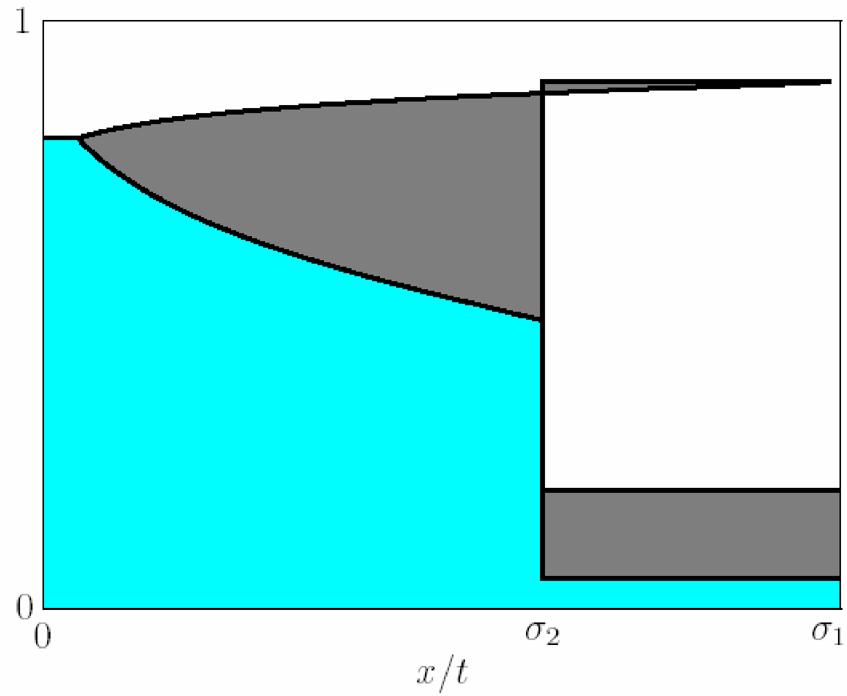




ROLE OF DETACHED BRANCHES

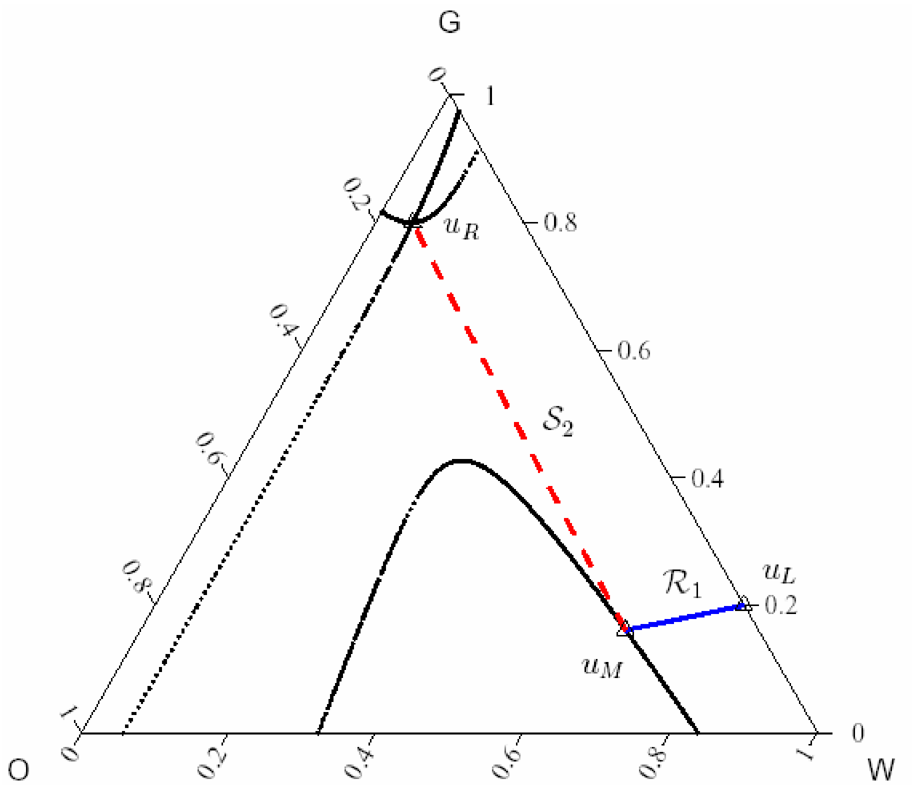
- **Inadmissible** solution involving **detached** branch: $s_1 > s_2^{\text{det}}$

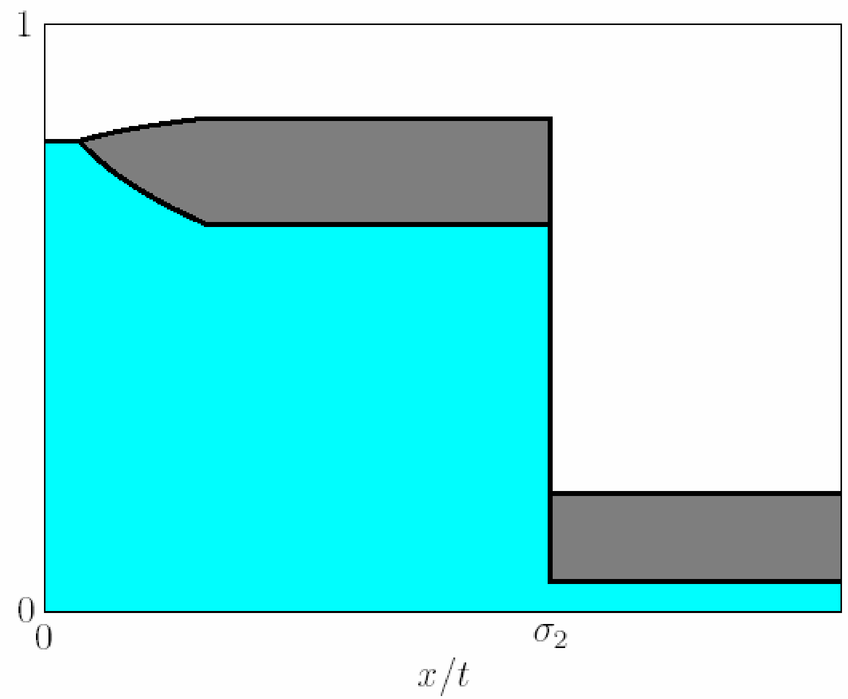




ROLE OF DETACHED BRANCHES

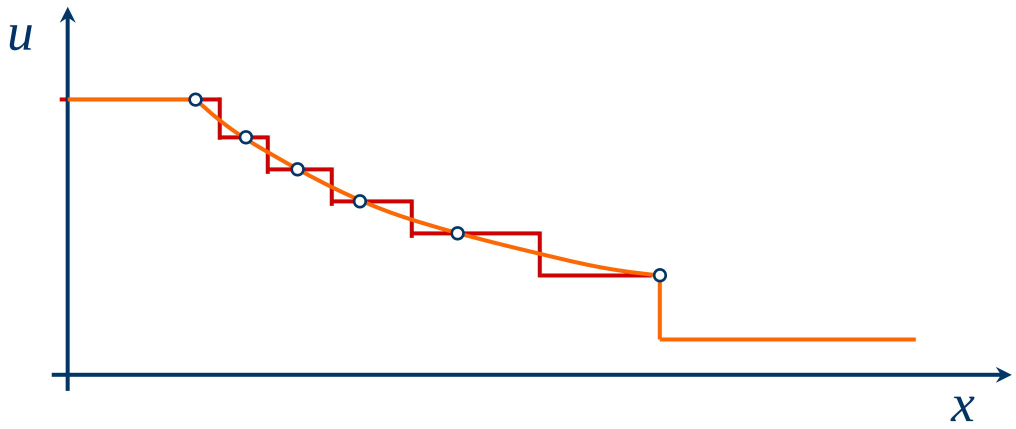
- **Admissible** solution involving **detached** branch: $S_1 < S_2^{det}$





FRONT-TRACKING IMPLEMENTATION

- If the solution involves discontinuities only, the front-tracking method is **exact**
- **Rarefactions** are approximated by a series of (small) jump discontinuities

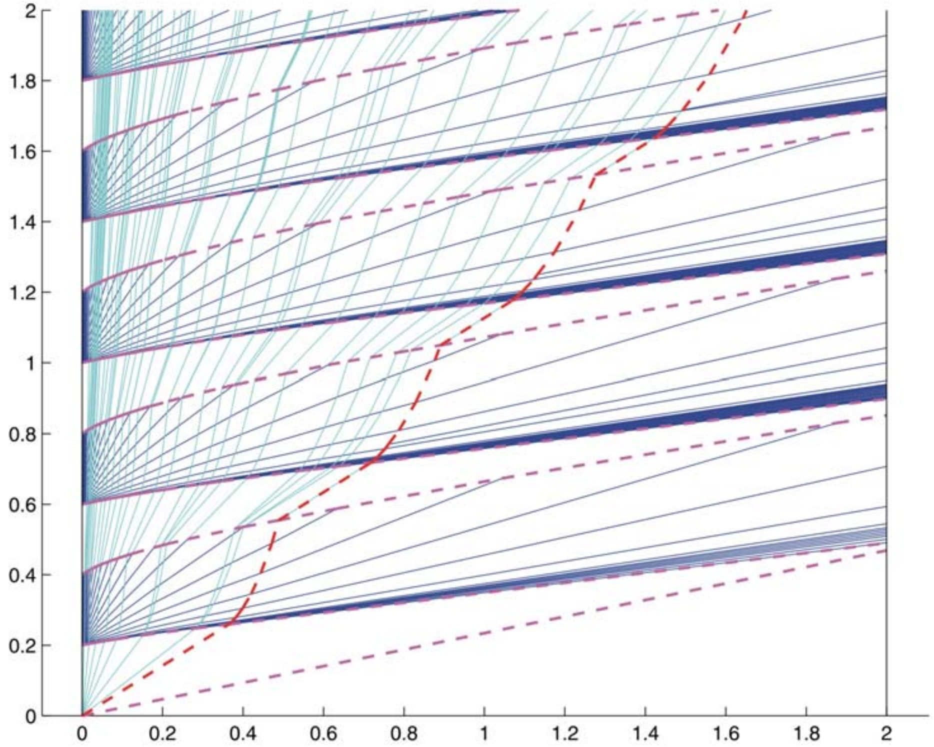


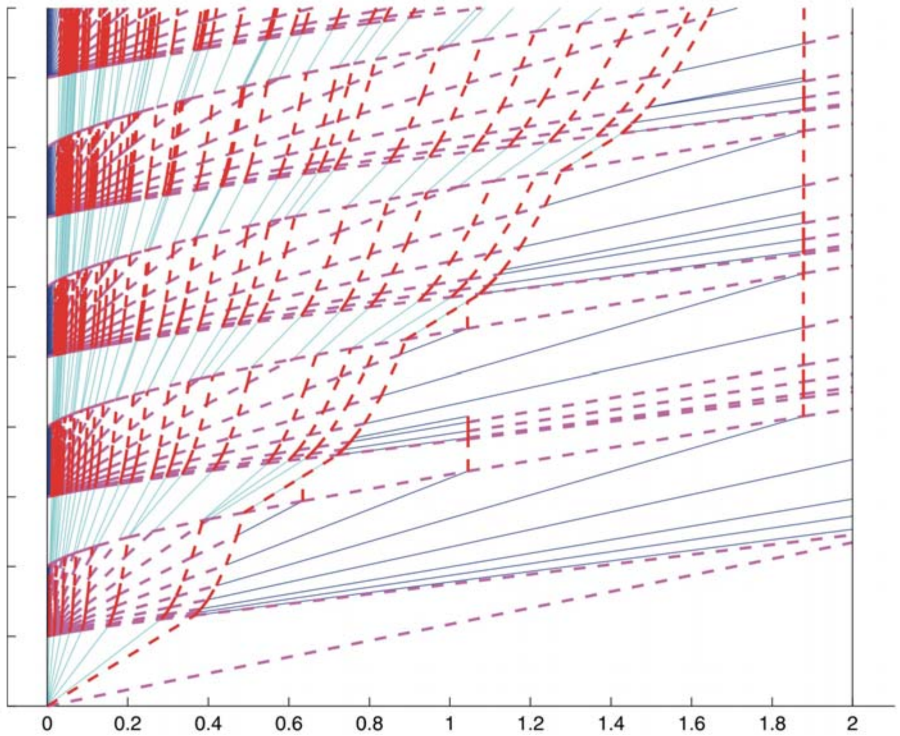
- **Data reduction:** Exceedingly small Riemann problems are discarded to avoid blow-up of number of discontinuities

DATA REDUCTION

1. If $|u_L - u_R| \le \delta_1$, ignore the Riemann problem
2. If $\delta_1 < |u_L - u_R| \le \delta_2$, approximate the Riemann problem by a single discontinuity with shock speed equal the average of the Rankine–Hugoniot velocity of each component
3. If $\delta_2 < |u_L - u_R| \le \delta_3$, approximate the Riemann problem by a two-shock solution $S_1 S_2$. If $\sigma_1 \not\le \sigma_2$, goto 4.
4. Otherwise solve the full Riemann problem

DATA REDUCTION





Left: full resolution of all wave interactions, 5563 in total

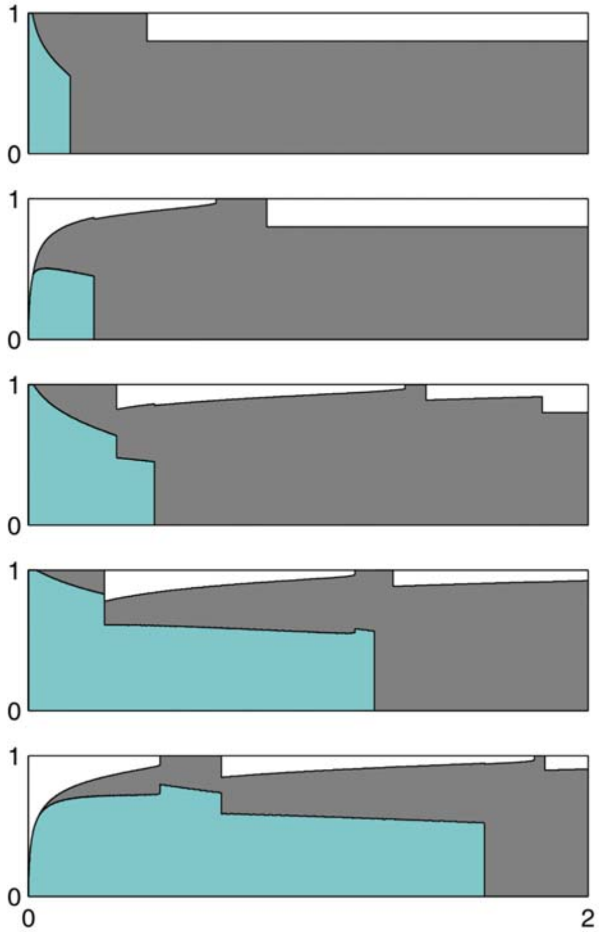
S1/S2: dashed red/magenta line

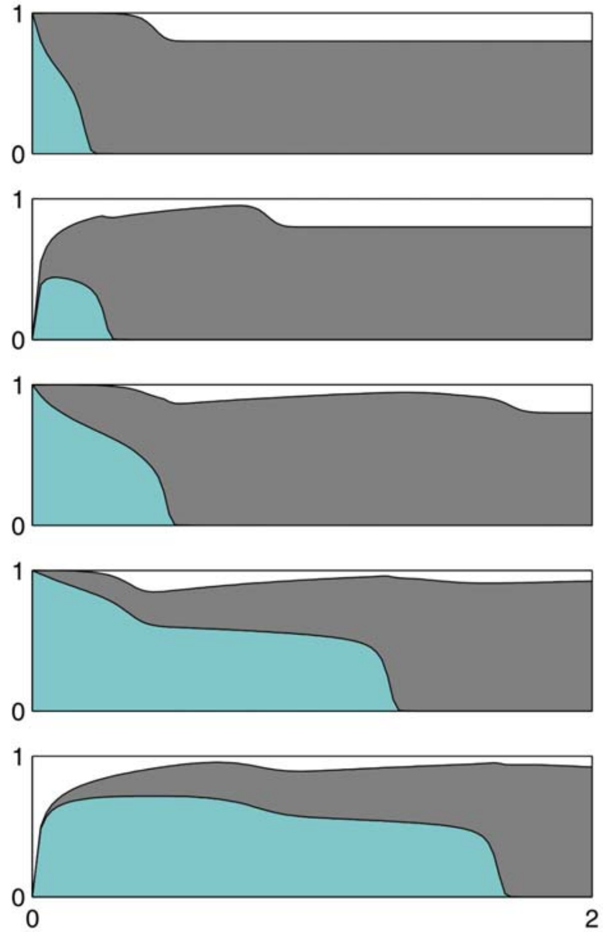
R1/R2: solid cyan/blue line

Right: weak wave interactions approximated by shocks, 1833 interactions in total of which 234 fully resolved

Ratio of runtimes is 4.6 : 1

COMPARISON WITH THE UPWIND FVM





Left: front-tracking solution consisting of 1.6 million Riemann problems.

Right: fully implicit upwind method with 100 grid cells and a Courant number of 1.0

The runtimes were approximately equal.

WATER SATURATION AFTER 2000 DAYS