Adaptive Multiscale Streamline Simulation and Inversion for High-Resolution Geomodels

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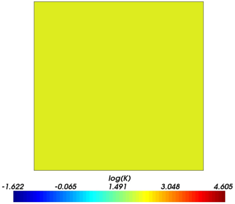
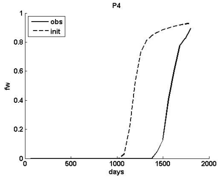
ECMOR XI, Bergen, September 8–11, 2008

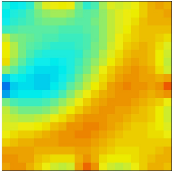
History matching is the procedure of modifying the reservoir description to match measured reservoir responses.

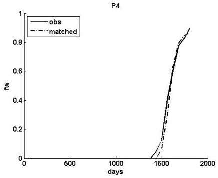
Initial:



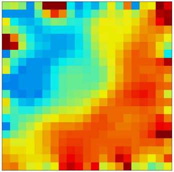


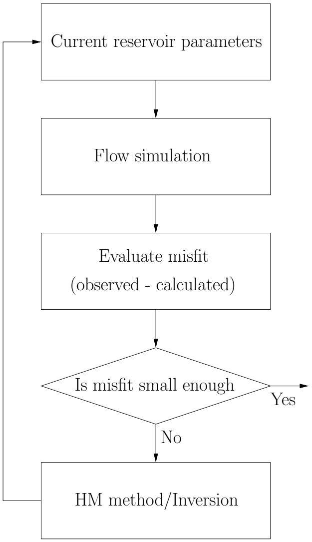
Matched:





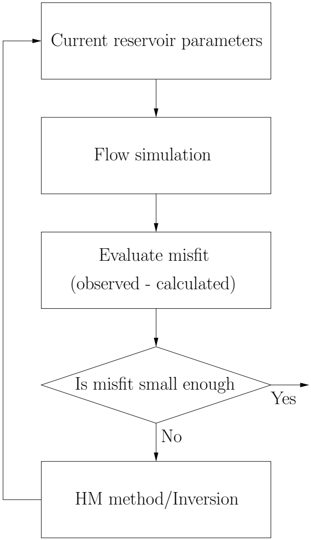
Reference:





$$E = \sum (\mathbf{d}^{\mathsf{obs}} - \mathbf{d}^{\mathsf{cal}})^2, \quad \mathbf{d}^{\mathsf{cal}} = g(\mathbf{m})$$

Challenges in history-matching loop



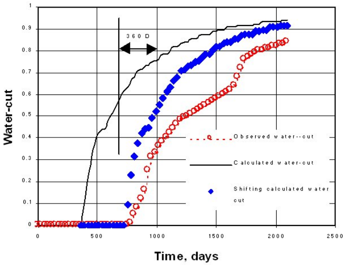
Problems:

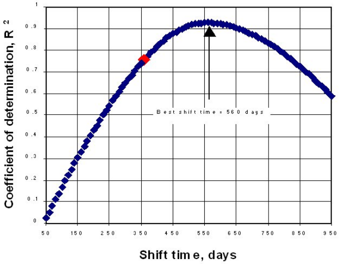
- highly under-determined problem $\longrightarrow$ non-uniqueness
- errors in model, data, and methods
- nonlinear forward model
- *non-convex misfit functions*
- *forward simulations are computationally demanding*

Inversion method: Generalized Travel-Time (GTT) inversion with analytic sensitivities [Vasco et al. (1999), He et al. (2002)]

The generalized travel time is defined as the 'optimal' time–shift that maximizes

$$R^2(\Delta t) = 1 - \frac{\sum[y^{\text{obs}}(t_i + \Delta t) - y^{\text{cal}}(t_i)]^2}{\sum[y^{\text{obs}}(t_i) - \bar{y}^{\text{obs}}(t_i)]^2}.$$





Basic underlying principles for the history–matching algorithm

- Minimize travel-time misfit for water–cut by iterative least-square minimization algorithm.
- Preserve geologic realism by keeping changes to prior geologic model minimal (if possible).
- Only allow smooth large-scale changes. Production data have low resolution and cannot be used to infer small-scale variations.

Minimization of functional:

$\Delta\tilde{\mathbf{t}}$: Travel–time shift
$\mathbf{S}$: Sensitivity matrix
$\mathbf{m}$: Reservoir parameters

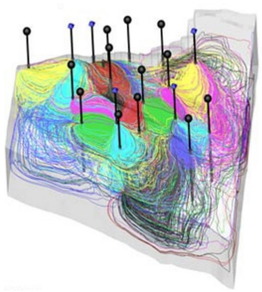
$$\|\Delta\tilde{\mathbf{t}} - \mathbf{S}\delta\mathbf{R}\| + \overbrace{\underbrace{\beta_1\|\delta\mathbf{R}\|}_{norm} + \underbrace{\beta_2\|\mathbf{L}\delta\mathbf{R}\|}_{smoothing}}^{Regularization}$$

$\mathbf{S}$ computed analytically along streamlines from a single flow simulation

Streamline-based history matching

Features of streamlines

- Very well suited for modeling large heterogeneous multiwell systems dominated by convection
- Generally fast flow simulation
- Delineate flow pattern (injector-producer pairs)
- Enables analytic sensitivities



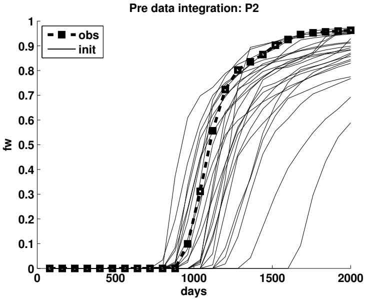
Source: www.techplot.com

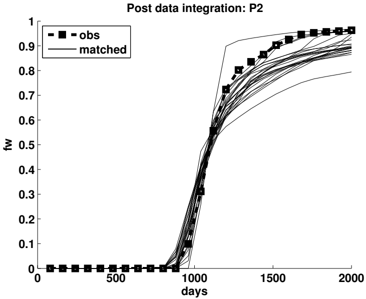
Classes of streamline-based history-matching methods

- Assisted history matching
- (Generalized) travel-time inversion methods
- Streamline-effective property methods
- Miscellaneous

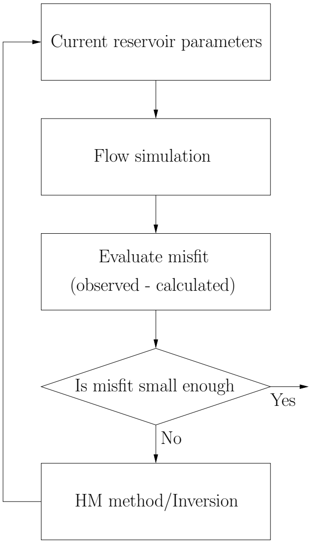
Example: Uncertainty quantification

Simple two-phase model (end-point mobility $M = 0.5$) on a 2D horizontal reservoir, 25×25 cells with lognormal permeability. Result after eight iterations:





Statistical analysis of mean and standard deviation



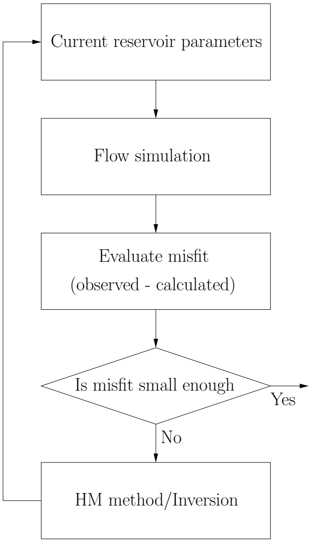
Streamline simulation much faster than conventional FD-methods.

Still, room for improvement.

Observations:

- pressure solver most expensive part of flow simulation
- parameters change very little from one flow simulation to the next

Challenge II: Long runtime for forward simulations



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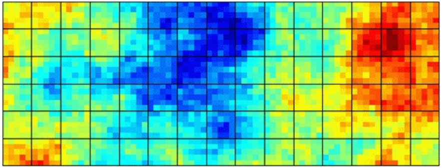
Idea: should reuse computations in areas with minor changes
$\longrightarrow$ multiscale methods

Multiscale pressure solver

Upscaling and downscaling in one step. Runtime like coarse-scale solver, resolution like fine-scale solver.

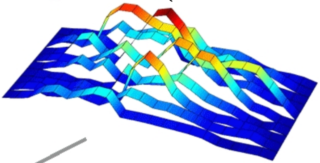
Fine grid: 75×30. Coarse grid: 15×6

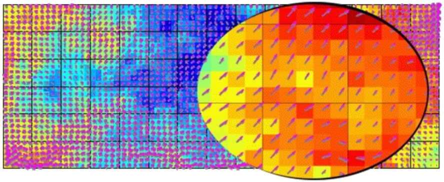


Basis functions for each pair of coarse blocks $T_i \cup T_j$:

$$\Psi_{ij} = -\lambda K \nabla \Phi_{ij}$$

$$\nabla \cdot \Psi_{ij} = \begin{cases} w_i(x), & x \in T_i \\ -w_j(x), & x \in T_j \end{cases}$$





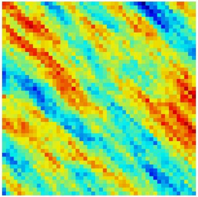
Global linear system with 249 unknowns:

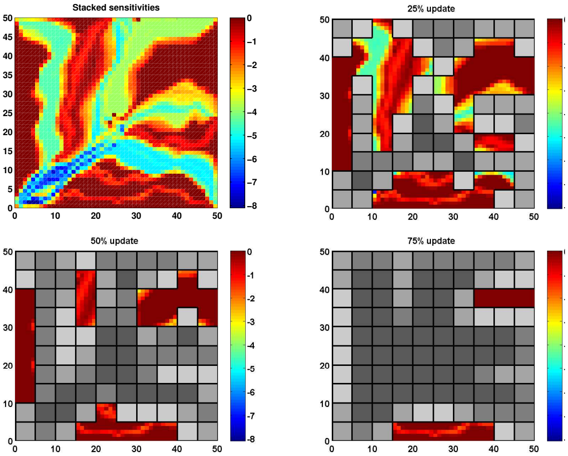
$$\nabla \cdot v = q, \quad v = -\lambda K \nabla p$$

Coarse grid: pressure and fluxes. Fine grid: fluxes

Further computational savings

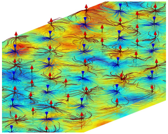
Can also reuse basis functions from previous forward simulation.
General idea: use sensitivities to steer updating





1 million cells, 32 injectors, and 69 producers
2475 days ≈ 7 years of water-cut data

Analytical sensitivities along streamlines + travel-time inversion (quasi-linearization of misfit functional)



Solver	Misfit			CPU-time (wall clock)		
	T	A	$\Delta \ln k$	Total	Pres.	Transp.
Initial	100.0	100.0	0.821	—	—	—
TPFA	8.9	53.5	0.806	64 min	33 min	28 min
Multiscale	11.2	47.3	0.812	43 min	7 min	32 min
Multiscale	10.4	45.4	0.828	17 min	7 min	6 min

Time-shift misfit: $\|\Delta \mathbf{t}\|_2$

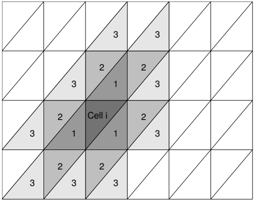
Amplitude misfit: $[\sum_k \sum_j (d_w^{\text{obs}} - d_w^{\text{cal}})^2]^{1/2}$

Permeability discrepancy: $\sum_{i=1}^{N} |\ln k_i^{\text{ref}} - \ln k_i^{\text{match}}|/N$

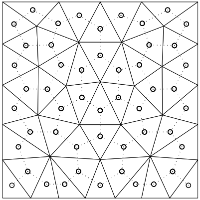
Generalized Laplacian stencil

$$L_i\,\mathbf{m} = -w_{ii}m_i + \sum_{j\in\mathcal{N}(i)} w_{ji}m_j,$$
$$w_{ji} = w_{\text{norm}}\cdot\rho(\zeta(i,j);R),\qquad w_{ii} = \sum_{j\in\mathcal{N}(i)} w_{ji}.$$

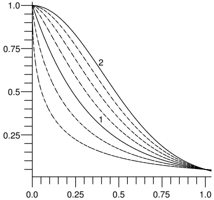
Neighborhood: $\mathcal{N}(i)$



Distance: $\zeta(i,j)$



Correlation: $\rho(\zeta;R,\ldots)$

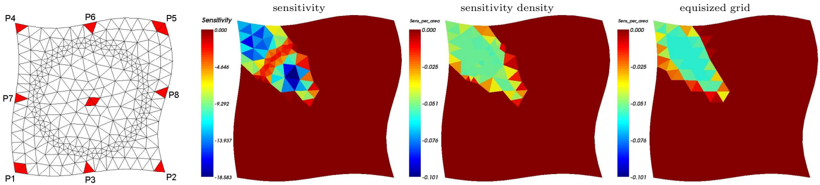


Properties of smoothing: independent of grid density, reduce to Laplacian on Cartesian grids, decay with distance $\zeta(i,j)$, be zero outside finite range, be bounded as $\zeta \to 0$.

Sensitivities are spatially additive.

$$\text{Small/large cells} \longrightarrow \text{small/large sensitivities}$$

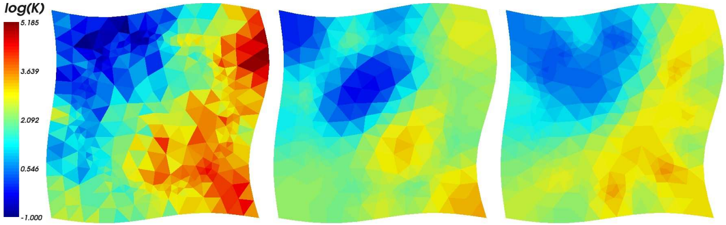
Thus, grid effects are to be expected

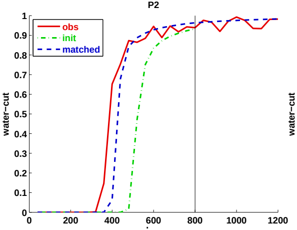


Rescaling of sensitivities:

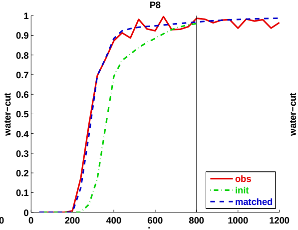
Permeability modification ΔK_i scales with sensitivity G_i.
Splitting cell in two $\Longrightarrow \Delta K \rightarrow\sim \frac{1}{2}\Delta K$ in each subcell
Therefore: scale sensitivity by relative volume, $\tilde{G}_i = G_i(\bar{V}/V_i)$

Example:
1200 days observed (5% noise added), 800 matched, 400 predicted

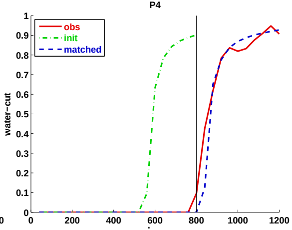




worst match



best match



least data

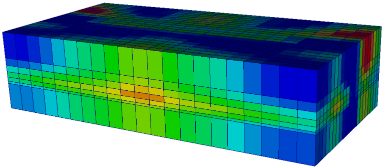
Synthetic test case:

$21 \times 21 \times 7$ tensor-product grid with layers of varying thickness.

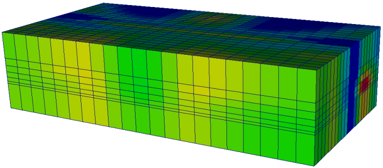
Layer	1	2	3	4	5	6	7
Thickness	1.0	0.089	0.164	0.212	0.251	0.285	1.0

Initial model: two different constant values

True model: homogeneous



K from original G_{ij}



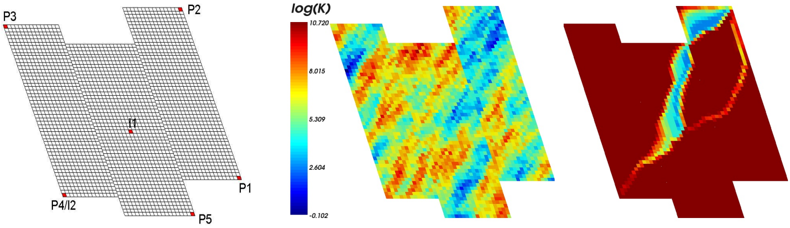
K obtained from scaled G_{ij}

Example:
Corner-point grid with two non-sealing strike-slip faults

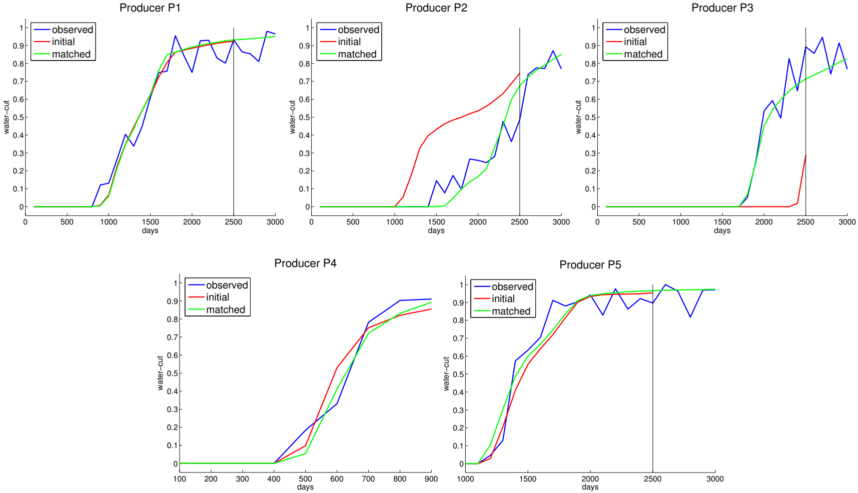
Infill drilling case:

Production data from 3000 days, 2500 used in history match.
At 900 days:

- new producer P5 drilled
- producer P4 converted to injector



Example:
Corner-point grid with two non-sealing strike-slip faults, cont'd



Extensions and similar ideas

Direct continuation of previous work:

- Unstructured grids (done for inversion algorithm)
- Corner-point grids (testing remains to be done on real models)
- Other types of data / more general flow

Other possible directions using streamlines:

- Closed-loop reservoir management
- History-matching seismic data
- Use of sensitivities for other optimization workflows
- ...